

# Physics

---

## Modeling Nature

*A Mastery-Oriented Curriculum*

**John D. Mays**



*Austin, Texas*  
2015

© 2015 NOVARE SCIENCE & MATH LLC

All rights reserved. Except as noted below, no part of this book may be reproduced or transmitted in any form or by any means, electronic or mechanical, including photocopying, recording, or by information storage and retrieval systems, without the written permission of the publisher, except by a reviewer who may quote brief passages in a review.

All images attributed to others under CC-BY-SA-3.0 or other Creative Commons licenses may be freely reproduced and distributed under the terms of those licenses.

Scriptural quotations are from The Holy Bible, English Standard Version, © 2001 by Crossway Bibles, a publishing ministry of Good News Publishers. Used by permission. All rights reserved.

Published by  
Novare Science & Math LLC  
P. O. Box 92934  
Austin, Texas 78709-2934  
novarescienceandmath.com



Printed in the United States of America

ISBN: 978-0-9863529-0-4

Cover design by Nada Orlic, <http://nadaorlic.info/>

For the complete catalog of textbooks and resources available from Novare Science and Math, visit [novarescienceandmath.com](http://novarescienceandmath.com).

# Contents

---

## Preface

<b>For Teachers</b>	xiv
Text Content and Student Preparedness	xiv
Emphasis on Mastery, Integration, and Kingdom	xvi
Recommendations for Teaching With This Text	xvii
Laboratory Work and Lab Reports	xx

## Preface

<b>For Students</b>	xxi
Why Physics is Challenging	xxi
Strategies for Making Physics Manageable	xxii
Studying for Mastery	xxiii

## Chapter 1

<b>Mathematical Tools</b>	2
1.1 Science and Measurements	4
1.1.1 No Measurements, No Science	4
1.1.2 Matter, Volume, and Mass	4
1.1.3 The SI Unit System	5
1.1.4 MKS Units	7
1.2 Uncertainty in Measurements	7
1.2.1 Error and Uncertainty	7
1.2.2 Distinguishing Between Accuracy and Precision	7
1.2.3 Significant Digits	8
1.2.4 Random and Systematic Error	12
1.2.5 Standard Deviation	13
1.2.6 Calculating Percent Difference	14
1.3 Modeling Nature	15
1.3.1 Science as Mental Model Building	15
1.3.2 Truth and Facts	16
1.3.3 Facts, Theories, Hypotheses, and Experiments	17
1.4 Vector Methods	19
1.4.1 Scalars and Vectors	19
1.4.2 How to Learn Vector Addition	21
1.4.3 Vector Addition—the Graphical Method	21
1.4.4 Additional Details About the Direction of a Vector	24
1.4.5 Vector Addition—the Trigonometric Method	25
1.4.6 Multiplying a Vector by a Scalar	28

1.4.7 Vector Multiplication		29
<i>Connections in Physics</i>	<i>Vector Fields</i>	33
Chapter 1 Exercises		34
<b>Chapter 2</b>		
<b>Uniform Motion</b>		38
2.1 How Physics Is Organized		40
2.2 Rectilinear Motion		40
2.2.1 The Terminology of Motion		40
2.2.2 Coordinate Systems		40
2.2.3 Velocity and Acceleration		40
<i>Connections to Calculus</i>	<i>Derivatives</i>	43
2.2.4 The Equations of Kinematics		45
<i>Connections in Physics</i>	<i>Correct Problem-Solving Method</i>	50
<i>Connections in Physics</i>	<i>Free Fall</i>	54
2.2.5 Symmetry in Kinematics Problems		56
2.3 Projectile Motion		57
2.3.1 Modeling Assumptions		57
2.3.2 Vector Analysis and Calculation Method		58
<i>Connections in Physics</i>	<i>Fictitious Forces</i>	61
2.4 Relative Velocity		64
Chapter 2 Exercises		69
<b>Chapter 3</b>		
<b>Forces, Fields, and Newton's Laws of Motion</b>		74
3.1 Forces and Fields: The Big Picture		76
3.1.1 The Four Fundamental Interactions		76
3.1.2 Fields		78
3.2 Historical Views on Motion		84
3.3 Newton's Laws of Motion		85
3.3.1 The Laws of Motion		86
<i>Connections in Physics</i>	<i>The Laws of Motion</i>	86
<i>Connections in Physics</i>	<i>Motion at Relativistic Speeds</i>	88
3.3.2 Mass and Weight		89
3.3.3 General Principles for Modeling Complex Systems		90
3.3.4 Free Body Diagrams and Force Components		91
3.4 Friction		97
3.4.1 The Source of Friction		97
3.4.2 Modeling Friction		98
3.4.3 Solving Dynamics Problems Involving Friction		101
<i>Connections to Calculus</i>	<i>The Second Law and Impulse</i>	106
Chapter 3 Exercises		107
<b>Chapter 4</b>		
<b>Static Equilibrium and Torque</b>		112
4.1 Static Equilibrium		114
4.1.1 Static Equilibrium		114
4.1.2 Two Conditions for Static Equilibrium		115
4.2 Modeling Torque		116
4.2.1 Torque as a Vector Quantity		116

4.2.2 Visualizing and Calculating Torque	116
4.3 Modeling Static Equilibrium	117
4.3.1 Free Body Diagrams for Static Equilibrium	117
4.3.2 Pivot Points and Torque Computations	121
4.3.3 Solving Problems in Static Equilibrium	123
Chapter 4 Exercises	130
<b>Chapter 5</b>	
<b>Energy</b>	134
5.1 Defining Energy	136
5.1.1 Energy, Matter, and the Beginning of the Universe	136
5.1.2 The Quantization of Energy	137
5.2 Forms of Energy	139
5.2.1 Units for Energy and Power	139
5.2.2 Forms of Energy	140
5.2.3 The Mechanical Equivalent of Heat	147
5.2.4 Energy Loss in Mechanical Systems	149
5.3 The Conservation of Mass-Energy	149
5.3.1 The Energy Fundamentals	149
5.3.2 The Law of Conservation of Energy	150
5.3.3 Conservative Fields	153
5.3.4 The Work-Energy Theorem	154
<i>Connections to Calculus</i> <i>Energy Rates of Change</i>	155
5.3.5 Einstein and Mass-Energy Equivalence	160
<i>Connections in Physics</i> <i>The Photoelectric Effect</i>	162
Chapter 5 Exercises	163
<b>Chapter 6</b>	
<b>Momentum</b>	168
6.1 Momentum and Impulse	170
6.1.1 Momentum	170
6.1.2 Impulse and Newton's Second Law	170
6.2 Conservation of Momentum	172
6.2.1 Conservation Laws in Physics	172
6.2.2 The Law of Conservation of Momentum	173
6.2.3 Conservation of Momentum and Newton's Laws of Motion	177
6.3 Elastic and Inelastic Collisions	177
6.3.1 Collisions and Energy Loss in Systems	177
6.3.2 Combining Momentum and Energy	178
<i>Connections in Physics</i> <i>Discovery of the Neutron</i>	179
<i>Connections in Physics</i> <i>Compton Scattering</i>	182
Chapter 6 Exercises	183
<b>Chapter 7</b>	
<b>Rotating Systems</b>	186
7.1 Angular Quantities	188
7.1.1 Radian Measure	188
7.1.2 Angular Displacement, Angular Velocity, and Angular Acceleration	189
7.1.3 Tangential Quantities	193

## Contents

7.2	Torque and Angular Acceleration	196
7.2.1	Moment of Inertia	196
7.2.2	Newton's Second Law for Rotation	197
7.3	Centripetal Force	198
7.3.1	Motion in a Circle	198
7.3.2	Calculations With Centripetal Force	199
	<i>Connections to Calculus</i>	<i>Moment of Inertia</i>
7.4	Newton's Law of Universal Gravitation	204
	<i>Connections in Physics</i>	<i>Henry Cavendish and G</i>
	Chapter 7 Exercises	208
<b>Chapter 8</b>		
<b>Energy and Momentum in Rotating Systems</b>		
8.1	Rotational Kinetic Energy	214
8.1.1	More Translational-Rotational Parallels	214
8.1.2	Rotational Kinetic Energy as an Energy Storage Medium	216
8.1.3	Conservation of Energy with Rotating Systems	219
8.2	Combining Rotational and Translational Kinetic Energy	220
8.3	Angular Momentum	221
	<i>Connections in Physics</i>	<i>Quantization of Angular Momentum</i>
	<i>Connections in Physics</i>	<i>Precession</i>
	Chapter 8 Exercises	225
<b>Chapter 9</b>		
<b>Pressure and Buoyancy</b>		
9.1	Pressure	232
9.1.1	Pressure as Force Applied to an Area	232
9.1.2	Pressure In a Fluid Medium	233
9.1.3	Air Pressure	234
9.1.4	Absolute Pressure and Gauge Pressure	237
9.1.5	Pascal's Law	240
9.1.6	Bernoulli's Principle	242
9.2	Buoyancy and Archimedes' Principle	244
9.2.1	Buoyancy	244
9.2.2	Archimedes' Principle	244
9.2.3	Flotation	248
	Chapter 9 Exercises	249
<b>Chapter 10</b>		
<b>Gases, Kinetic Theory, and Heat</b>		
10.1	Moles and Molar Masses	256
10.1.1	Moles and the Avogadro Constant	256
10.1.2	Molar Mass	256
10.1.3	Gram Masses of Atoms and Molecules	257
10.2	The Gas Laws	259
10.2.1	Boyle's Law	259
10.2.2	Charles' Law	260
10.2.3	Avogadro's Law	263
10.2.4	The Ideal Gas Law	263

10.3	The Kinetic-Molecular Theory of Gases	269
10.3.1	Velocity Distribution of Gases	270
	<i>Connections in Physics</i> <i>Fundamental Constants in Nature</i>	271
10.3.2	The Kinetic-Molecular Theory of Gases	272
10.3.3	Temperature and Molecular Energy	273
10.4	Thermal Properties of Matter	277
10.4.1	States of Matter	277
10.4.2	Heat Transfer	277
10.4.3	Phase Transitions and Phase Diagrams	278
10.4.4	Thermal Properties of Matter	280
10.4.5	Calorimetry	283
10.4.6	Evaporation	287
10.4.7	Vapor Pressure	287
	Chapter 10 Exercises	288
	<b>Chapter 11</b>	
	<b>Thermodynamics</b>	292
11.1	The First Law of Thermodynamics	294
11.1.1	The Ideal Gas System Model	294
11.1.2	The First Law of Thermodynamics	295
11.1.3	State Variables and Thermodynamic States	296
11.1.4	Work and Thermodynamic States	297
	<i>Connections to Calculus</i> <i>Work as Area Under the PV Curve</i>	299
11.2	Thermodynamic Processes	300
11.2.1	Thermodynamic Processes	300
11.2.2	Qualitative First Law Applications	303
11.3	The Second Law of Thermodynamics	304
11.3.1	Disorder and Directionality	304
11.3.2	Microstates, Entropy, and Reversibility	305
11.3.3	Heat Engines and Maximum Theoretical Efficiency	308
11.3.4	Refrigeration	311
	Chapter 11 Exercises	313
	<b>Chapter 12</b>	
	<b>Simple Harmonic Motion, Waves, and Sound</b>	316
12.1	Simple Harmonic Motion	318
12.1.1	Modeling Oscillation	318
12.1.2	Simple Harmonic Motion	319
	<i>Connections to Calculus</i> <i>Simple Harmonic Motion</i>	321
12.1.3	Energy In Simple Harmonic Motion	322
12.1.4	The Simple Pendulum and the Small-Angle Approximation	324
12.2	Wave Modeling and Interactions	326
12.2.1	Modeling Waves	326
12.2.2	Reflection, Refraction, and Dispersion	328
12.2.3	Standing Waves and Resonance	331
12.2.4	Diffraction and Interference	334
12.3	Sound	338
12.3.1	Sound Intensity and the Inverse Square Law	338
12.3.2	Sound Pressure Level	340

12.3.3 Frequency Response	345
12.3.4 The Doppler Effect	346
12.3.5 The Speed of Sound	349
Chapter 12 Exercises	351
<b>Chapter 13</b>	
<b>Electrostatics and Electric Circuits</b>	<b>356</b>
13.1 Charge and Static Electricity	358
13.1.1 Charge	358
13.1.2 Static Electricity	358
13.2 Conservation of Charge and Coulomb's Law	359
13.2.1 Conservation of Charge	359
<i>Connections in Physics</i>	<i>Feynman Diagrams</i>
	360
13.2.2 Coulomb's Law	361
13.3 Electric Fields and Charge Symmetries	364
13.3.1 Spherical Field Symmetry	364
13.3.2 Cylindrical Field Symmetry	364
13.3.3 Planar Field Symmetry	365
13.4 Energy in Electric Fields	368
13.4.1 Work, Potential, and Voltage	368
<i>Connections to Calculus</i>	<i>Absolute Potential</i>
	370
13.4.2 Potential and Equipotentials	372
13.4.3 Capacitors	373
13.4.4 Energy Storage in Capacitors	376
13.5 DC Circuits	377
13.5.1 Electric Circuits	377
13.5.2 Resistance in Conductors	382
13.5.3 Resistor Combinations and Equivalent Resistance	383
13.5.4 Kirchhoff's Laws	389
13.5.5 DC Circuit Analysis	392
Chapter 13 Exercises	397
<b>Chapter 14</b>	
<b>Magnetism and Time-Varying Circuits</b>	<b>404</b>
14.1 Forces Caused by Magnetic Fields	406
14.1.1 Fields and Flux	406
14.1.2 Magnetic Forces on Wires and Ampère's Law	408
<i>Connections to Calculus</i>	<i>Magnetic Flux and the B-Field</i>
	409
14.1.3 Forces on Moving Charges	413
14.1.4 Torque on a Current Loop and the DC Motor	416
<i>Connections in Physics</i>	<i>Bubble Chambers</i>
	417
14.2 Faraday's Law, Generators, and Transformers	420
14.2.1 Faraday's Law of Magnetic Induction	420
<i>Connections to Calculus</i>	<i>Faraday's Law of Magnetic Induction</i>
	421
14.2.2 Generators	422
14.2.3 Transformers	423
14.3 Inductance and Time-Varying Circuits	427
14.3.1 Inductance	427
14.3.2 Steady-State AC Circuits	430



14.3.3 RC Circuits		433
<i>Connections to Calculus</i>	<i>Step Response of a Capacitor</i>	435
14.3.4 RL Circuits		437
14.3.5 RLC Circuits		438
14.4 Lenz's Law		440
14.4.1 Lenz's Law		440
14.4.2 Back-EMF		443
Chapter 14 Exercises		443
<b>Chapter 15</b>		
<b>Geometric Optics: A Brief Introduction</b>		448
15.1 Ray Optics		450
15.1.1 Light As Rays		450
15.1.2 Human Image Perception		450
15.1.3 Flat Mirrors and Ray Diagrams		450
15.1.4 Real and Virtual Images		452
15.2 Optics and Curved Mirrors		452
15.2.1 Concave and Convex Optics		452
15.2.2 Approximations in Geometric Optics		453
15.2.3 Spherical Mirrors		454
15.2.4 The Mirror Equation		455
15.3 Lenses		457
15.3.1 Light Through a Lens		457
15.3.2 Single-Lens Applications		459
15.3.3 The Lens Equation		461
15.3.4 Multiple-Lens Systems		463
<i>Connections in Physics</i>	<i>Rainbows</i>	467
15.3.5 Imaging with the Eye		468
Chapter 15 Exercises		468
<b>Chapter 16</b>		
<b>Nuclear Physics: A Brief Introduction</b>		470
16.1 Nuclides and Isotopes		472
16.1.1 Isotopes		472
16.1.2 Atomic Mass Unit		472
16.1.3 Nuclear Size and Density		473
16.1.4 Binding Energy and Mass Defect		474
16.1.5 Nuclear Equations		477
16.2 Radioactivity		477
16.2.1 Nuclear Stability		477
16.2.2 Nuclear Decay and Decay Series		478
16.2.3 Nuclear Half-Life		484
16.2.4 Radiometric Dating		488
16.3 Fission and Fusion Reactions		491
16.3.1 Nuclear Fission		491
<i>Connections in Physics</i>	<i>The Beginning of Nuclear Power</i>	493
16.3.2 Nuclear Fusion		495
Chapter 16 Exercises		498
<b>Glossary</b>		500

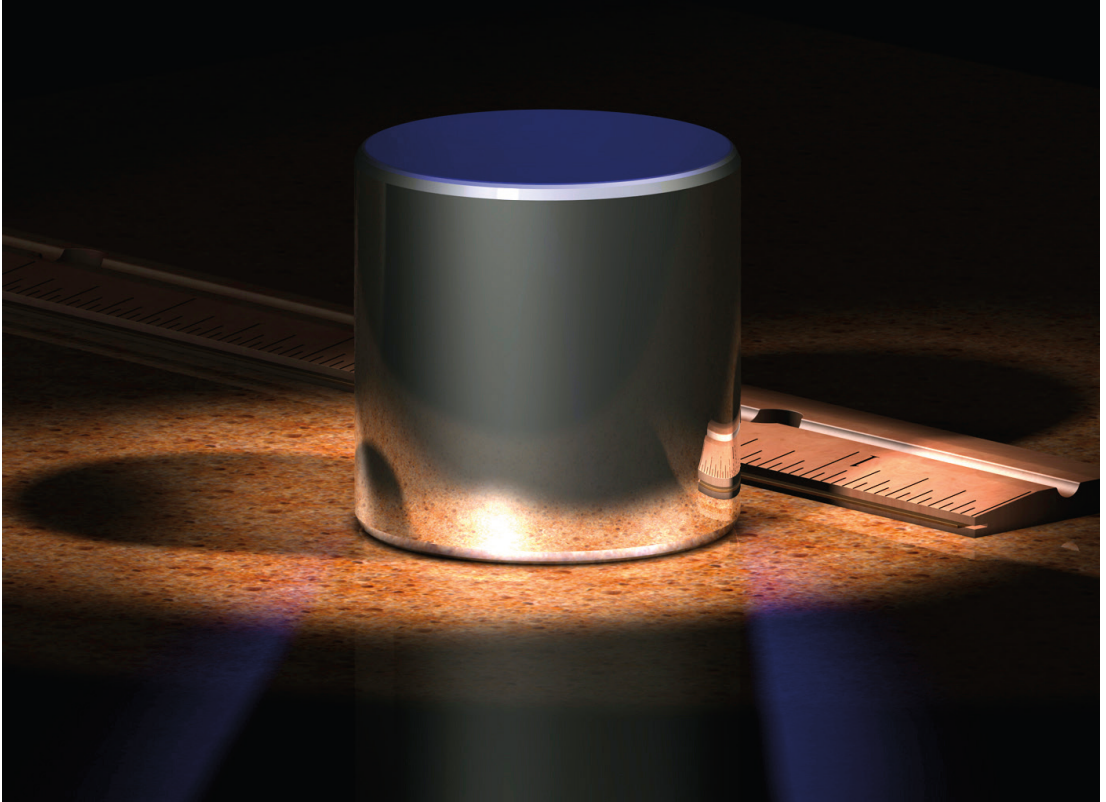
## Contents

Answers to Selected Exercises	515
Appendix A	522
Reference Data	522
Appendix B	526
Unit Conversions Tutorial	526
B.1 Basic Principles of Unit Conversion Factors	526
B.2 Tips for Converting Units of Measure	528
B.3 Converting Temperature Units	531
References and Citations	533
Image Credits	534
Index	537

# Chapter 1

---

## Mathematical Tools



The image above shows a small cylinder made of 90% platinum (element 78). This image is computer-generated, but it is an accurate representation of the official one-kilogram mass known as the International Prototype of the Kilogram (IPK). The IPK is maintained in a vault in Sèvres, France by the International Bureau of Weights and Measures. The mass of the platinum cylinder in that vault is the official definition of the kilogram.

The kilogram is the only one of the seven base units in the metric system that is still defined by an artifact (a man-made physical object). The others are now defined in terms of various constants found in nature. Officials are looking to change the definition of the kilogram so that it, too, is defined in terms of physical constants instead of an artifact. In 2014, a decision was made to explore new methods for defining the kilogram and research on various methods is now underway.

The kilogram is also the only base unit in the metric system that includes a metric prefix in its name.

## Objectives for Chapter 1

---

After studying this chapter and completing the exercises, you should be able to do each of the following tasks, using supporting terms and principles as necessary.

### SECTION 1.1

1. Distinguish between matter and mass.
2. Use standard SI units and prefixes correctly and consistently.
3. Describe the origin of the SI System and state the seven SI base units.
4. Explain what a derived unit is and give examples.
5. Explain what MKS units are and why we use them when solving physics problems.

### SECTION 1.2

6. Explain why all measurements contain uncertainty.
7. Distinguish between accuracy and precision.
8. Use significant digits properly when making measurements, interpreting measurements, and performing calculations.
9. Distinguish between random error and systematic error.
10. Describe the meaning of the sample standard deviation.
11. State the equation for the percent difference calculation used in evaluating experimental data.

### SECTION 1.3

12. Define *fact*, *theory*, *hypothesis*, and *experiment*.
13. Describe the relationships between facts, theories, hypotheses, and experiments and the roles these play in the process of scientific inquiry.

### SECTION 1.4

14. Describe the difference between vector and scalar quantities, and give several examples of each.
15. Use the properties of vectors to re-express vector quantities with negative directions or negative angles.
16. Multiply vectors by scalars.
17. Add vector quantities using graphical methods.
18. Calculate rectangular vector components, calculate the magnitude and direction of a vector from its  $x$ - and  $y$ -components, and use trigonometric methods to calculate vector sums.
19. Compute scalar (dot) and vector (cross) products.<sup>1</sup>
20. Use the right-hand rule to determine the direction of a vector product.

---

<sup>1</sup> This objective may be deferred until needed in Chapters 4 and 5.

## 1.1 Science and Measurements

### 1.1.1 No Measurements, No Science

One of the things that distinguishes scientific research from other fields of study is the central role played in science by *measurement*. In every branch of science, researchers study the natural world, and they do it by making measurements. The measurements we make in science are the data we use to quantify the facts we have and to test new hypotheses. These data—our measurements—answer questions such as What is its volume? How fast is it moving? What is its mass? How much time did it take? What is its diameter? What was its frequency and wavelength? When do we expect it to occur again? and many others. Without measurements, modern science would not exist.

Units of measure are crucial in science. Since science is so deeply involved in making measurements, you will be working with measurements a lot in this course. The value of a measurement is always accompanied by the units of measure—a measurement without its units of measure is a meaningless number. For this reason, your answers to computations in scientific calculations must always show appropriate units.

Before moving on to review units of measure, we will review the concepts of matter, volume, and mass.

### 1.1.2 Matter, Volume, and Mass

The best way to understand mass is to begin with *matter* and its properties. The term *matter* refers to anything composed of atoms or parts of atoms. Your thoughts, your soul, and your favorite song are not matter. You can write down your thoughts in ink, which *is* matter, and your song can be recorded onto a CD, which consists of matter. But ideas and souls are not material and are not made of what we call matter. Another part of this world that is not matter is *electromagnetic radiation*—light, radio waves, x-rays, and all other forms of electromagnetic radiation. Light is a form of energy; it is not matter and it has no mass.

All forms of matter may be described in terms of their physical and chemical properties. Here we will focus on just two: all matter takes up space, and all matter has inertia. Describing and comparing these two properties will help make clear what we mean by the term *mass*.

All matter takes up space. Even individual atoms and protons inside of atoms take up space. We *quantify* the amount space occupied by an object by specifying its *volume*. To say that the volume of an object is  $1,350\text{ cm}^3$  (the volume of a typical adult human brain) is to say that to fill a hollow shell this size would require 1,350 neatly stacked cubes, each with a volume of  $1\text{ cm}^3$ , illustrated in Figure 1.1.

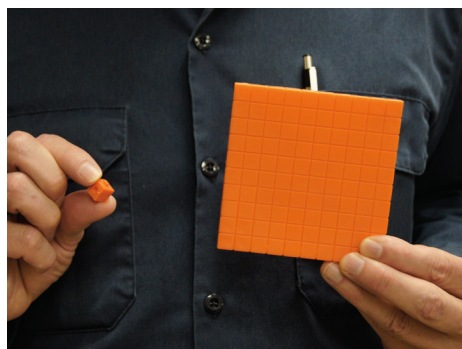


Figure 1.1 Volumes of  $1\text{ cm}^3$  (left) and  $100\text{ cm}^3$  (right).

All matter possesses the property of *inertia*. The effect of this property is that objects resist being accelerated. The more inertia an object has, the more difficult it is to accelerate the object. For example, if the inertia of an object is small, as with a golf ball, the object will be easy to accelerate. Golf balls are easy to throw, and if you hit one with a golf club it will accelerate at a high rate to a very high speed. But if the amount of inertia an object has is large, as with say, a grand piano, the object will be difficult to accelerate. Just try throwing a grand piano or hitting one with a golf club and you will see that it doesn't accelerate at all. This is because the piano has a great deal more inertia than a golf ball.

As with the property of taking up space, we need a way to quantify the inertia of an object. The way we do this is with the variable we call *mass*. The mass of an object is a numerical measurement specifying the amount of inertia the object has. Since inertia is a property of matter, and since all matter is composed of atoms, it should be pretty obvious that the more atoms there are packed into an object, the more mass it will have. And since the different types of atoms themselves have different masses, an object composed of more massive atoms will have more mass than an object composed of an equal number of less massive atoms.

The basic unit of measure we use to specify an object's mass is the *kilogram*. There are other units such as the gram and the microgram. The kilogram (kg) is one of the base units in the SI unit system, our topic in the next section.

We have established that the mass of an object is a measure of its inertia, which in turn depends on how many atoms it is composed of and how massive those atoms are. The implication of this is that an object's mass does not depend on where it is. A golf ball on the earth has the same mass as a golf ball at the bottom of the ocean, on the moon, or in outer space. Even where there is no gravity, the mass of the golf ball will be the same. This is what distinguishes the *mass* of an object from its *weight*.

Weight is caused by the force of gravity acting on an object composed of matter (which we often simply refer to as *a mass*). The weight of an object depends on where it is. An object—or mass—on the moon only weighs about 1/6 of its weight on earth, and in outer space, where there is no gravity, a mass has no weight at all. But the inertia of an object—and thus its mass—does not depend on where it is. This is because an object's mass is based on the matter the object is made of. A Steinway concert grand piano weighs 990 pounds on earth. In outer space, it weighs nothing and will float right in front of you. On the moon it weighs only 165 pounds—about as much as a slim man. But even in deep space, if you try to heave the grand piano—that is, try to accelerate it—the force you feel on your hands will be the same as it would be on the earth or on the moon. This is because the force you feel when you accelerate an object depends on the object's mass.

To summarize in different terms, inertia is a *quality* of all matter; mass is the *quantity* of a specific portion of matter. Inertia is a quality or property all matter possesses. Mass is a quantitative variable, and it specifies a quantity of matter.

### 1.1.3 The SI Unit System

The *U.S. Customary System* (USCS) is a system of units familiar to everyone raised in the United States. As familiar as this system is with its feet, pounds, and degrees Fahrenheit, this system is not used for scientific work. Thus without further ado, we will move on to the SI unit system.

The measurement system universally used for scientific work is the *International System of Units*, known as the *SI system* or the metric system. This system was published in 1960 but originated in France during the French Revolution. The original system included only the meter and the kilogram. Over the years, as measurement treaties were signed and scientific learning advanced, the system grew into the formal SI System that has now been in use since 1960. The SI System is administered by an organization in Sèvres, France (near Paris) known as the International Bureau of Weights and Measures. The SI System has been adopted almost globally. There

Unit	Symbol	Quantity
meter	m	length
kilogram	kg	mass
second	s	time
ampere	A	electric current
kelvin	K	temperature
candela	Cd	luminous intensity
mole	mol	amount of substance

Table 1.1. The seven base units in the SI unit system.

Unit	Symbol	Quantity
joule	J	energy
newton	N	force
cubic meter	m <sup>3</sup>	volume
watt	W	power
pascal	Pa	pressure

Table 1.2. Some SI System derived units.

are only three nations in the world that have not accepted the SI System as their official system of measurement: Myanmar, Liberia, and the United States. But even though our road sign markers still give distances in miles, in scientific work the SI System is the one we use.

There are seven *base units* in the SI System, listed in Table 1.1. All other SI units of measure, such as the joule (J) for measuring quantities of energy and the newton (N) for measuring

amounts of force, are based on these seven base units. Units based on combinations of the seven base units are called *derived units*. A few common derived units are listed in Table 1.2.

To accommodate measurements of vastly differing size, the SI System uses multipliers on the units to multiply them for large quantities, or to scale them down for smaller quantities. These multipliers are the *metric prefixes*. The complete list of the 20 official SI prefixes is in Table 1.3. You do not need to memorize all of these; some are rarely used. But you do need to memorize

Multiples	Prefix	deca-	hecto-	kilo-	mega-	giga-	tera-	peta-	exa-	zetta-	yotta-
	Symbol	da	h	k	M	G	T	P	E	Z	Y
	Factor	10	10 <sup>2</sup>	10 <sup>3</sup>	10 <sup>6</sup>	10 <sup>9</sup>	10 <sup>12</sup>	10 <sup>15</sup>	10 <sup>18</sup>	10 <sup>21</sup>	10 <sup>24</sup>
Fractions	Prefix	deci-	centi-	milli-	micro-	nano-	pico-	femto-	atto-	zepto-	yocto-
	Symbol	d	c	m	μ	n	p	f	a	z	y
	Factor	1/10	1/10 <sup>2</sup>	1/10 <sup>3</sup>	1/10 <sup>6</sup>	1/10 <sup>9</sup>	1/10 <sup>12</sup>	1/10 <sup>15</sup>	1/10 <sup>18</sup>	1/10 <sup>21</sup>	1/10 <sup>24</sup>

Table 1.3. The SI System prefixes.

some of them. I recommend that all high school science students commit to memory the prefixes listed in Table 1.4.

I will conclude this section with a few brief notes. First, when using the prefixes for quantities of mass, prefixes are never added to the kilogram. Prefixes are only added to the gram, even though the kilogram—not the gram—is the base unit in the SI system. Second, note that when writing the symbols for metric prefixes, the case of the letter matters: *kilo-* always takes a lower-case k, *mega-* always

Fractions			Multiples		
Prefix	Symbol	Factor	Prefix	Symbol	Factor
centi-	c	1/10 <sup>2</sup>	kilo-	k	10 <sup>3</sup>
milli-	m	1/10 <sup>3</sup>	mega-	M	10 <sup>6</sup>
micro-	μ	1/10 <sup>6</sup>	giga-	G	10 <sup>9</sup>
nano-	n	1/10 <sup>9</sup>	tera-	T	10 <sup>12</sup>
pico-	p	1/10 <sup>12</sup>			

Table 1.4. SI System prefixes to commit to memory.

takes an upper-case M, and so on. Third, one of the prefix symbols is not an English letter. The prefix μ for *micro-* is the lower-case Greek letter *mu*, the m in the Greek alphabet. Finally, pay close attention to the difference between multiple prefixes and fraction prefixes. Learning

to use the fraction prefixes properly is the most challenging part of mastering the SI System of units, and using them incorrectly in unit conversion factors is a common student error.



### 1.1.4 MKS Units

The study of physics is notorious for involving challenging problems. When you are dealing with complex problems involving lots of math, the last thing you want to do is fight your way through a host of unit prefixes and unit conversions to get all the units of measure to work out and agree. For this reason, when solving problems in physics we usually use a subset of the SI system units called *MKS system*. Using the MKS system means using only the SI base units such as the *meter*, the *kilogram*, and the *second* (hence, “MKS”) and the units derived directly from the base units.

The wonderful thing about solving problems in MKS units is that any calculation performed with MKS units will give a result in MKS units. This is why the MKS system is so handy and why we will use it almost exclusively. There are a few computations that are so simple that conversions to MKS units are not necessary, and I will point these out as we go. But for most of the problems you encounter in this text, you should always begin your problem solutions by converting all given quantities into MKS units. You will find many common conversion factors inside the back cover of the text and in Appendix A.

## 1.2 Uncertainty in Measurements

### 1.2.1 Error and Uncertainty

All measurements contain error because there is no such thing as an exact measurement or a perfect measurement instrument. Any measurement, if made with a precise enough instrument, will exhibit variation. For this reason, good experimental practice consists of performing measurements repeatedly so that the value under study consists not only of a single measurement but of an entire set of data. Scientists then communicate a measurement by specifying (usually) the mean value and a quantitative description of the *uncertainty* in the measurement. There are several ways to specify measurement uncertainty. The values in Table 1.5 are equivalent ways of expressing the mass of a proton, in units of  $10^{-27}$  kilograms, as it is known today. Note the space inserted after every third decimal place. This is common in values with more than six decimal places and makes them easier to read.

In the second of the values in the table (perhaps the most common way of specifying uncertainty), the value in parentheses (74) specifies the uncertainty in the last digit (the 7 in the billionths place). In the third value, the same degree of uncertainty is expressed in “parts per million.” To see this, note that one millionth of 1.672 621 777 is 0.000 001 673. Multiplying this by 0.044 gives us 0.044 of these millionths, which is 0.000 000 074. In other words, 0.000 000 074 is 0.044 millionths of 1.672 621 777.

Finally, note that these four expressions of uncertainty are *not* equivalent to saying that the true value of the proton mass is somewhere between  $1.672621851 \times 10^{-27}$  kg and  $1.672621703 \times 10^{-27}$  kg. They are statistical specifications relating to the amount of variation that occurs when scientists attempt to measure the proton mass. We will address this further in Section 1.2.5 below.

### 1.2.2 Distinguishing Between Accuracy and Precision

The terms *accuracy* and *precision* refer to the practical limitations inherent in making measurements. Science is all about investigating nature, and to do that we must make measurements.

1.672 621 777 ± 0.000 000 074
1.672 621 777(74)
1.672 621 777 ± 0.044 ppm

Table 1.5. Three equivalent ways of expressing the same uncertainty.



*Accuracy* relates to error—that is, to the lack of it—which is the difference between a measured value and the true value. The lower the error is in a measurement, the better the accuracy. Error can arise from many different sources including human mistakes, malfunctioning equipment, incorrectly calibrated instruments, vibrations, changes in temperature or humidity, or unknown causes that are influencing a measurement without the knowledge of the experimenter.

*Precision* refers to the resolution or degree of “fine-ness” in a measurement. The limit to the precision that can be obtained in a measurement is ultimately dependent on the instrument being used to make the measurement. If you want greater precision, you must use a more precise instrument. The degree of precision in every measurement is signified by the measurement value itself because the precision is a built-in part of the measurement. The precision of a measurement is indicated by the number of significant digits (or significant figures) included in the measurement value when the measurement is written down (see below).

Here is an example that illustrates the idea of precision and also helps to distinguish between precision and accuracy. Figure 1.2 is a photograph of a machinist’s rule and an architect’s scale placed one above the other. Since the marks on the two scales line up consistently, *these two scales are equally accurate*. But the machinist’s rule (on top) is more precise. The architect’s scale

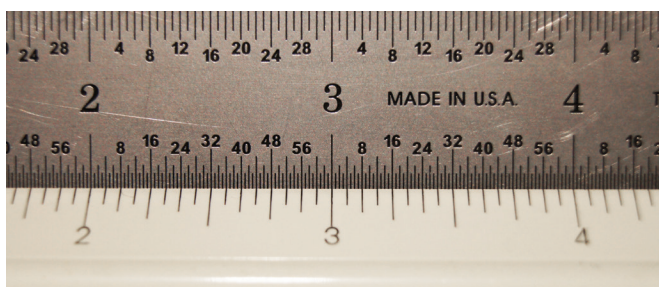


Figure 1.2. The accuracy of these two scales is the same, but the machinist’s rule (above) is more precise than the architect’s scale (below).

is marked in 1/16-inch increments, but the machinist’s rule is marked in 1/64-inch increments. Thus, *the machinist’s rule is more precise*.

It is important that you are able to distinguish between accuracy and precision. Here is another example to help illustrate the difference. Let’s say Theodore and Marius each buy digital thermometers for their homes. The thermometer Theodore buys costs \$10 and measures to the nearest 1°F. Marius pays \$40 and gets one that reads to

the nearest 0.1°F. Theodore reads the directions and properly installs the sensor for his new thermometer in the shade. Marius doesn’t read the directions and mounts his sensor in the direct sunlight, which causes a significant error in the thermometer reading when the sun is shining on it; thus Marius’ measurements are not very accurate. The result will be that Theodore has lower-precision, higher-accuracy measurements!

### 1.2.3 Significant Digits

The precision in any measurement is indicated by the number of *significant digits* it contains. Thus, the number of digits we write in any measurement we deal with in science is very important. The number of digits is meaningful because it shows the precision that was present in the instrument used to make the measurement.

Let’s say you are working a computational exercise in a science book. The problem tells you that a person is going to drive a distance of 110 miles at an average speed of 55 miles per hour and wants you to calculate how long the trip will take. The correct answer to this problem *will be different* from the correct answer to a similar problem with given values of 110.0 miles and 55.0 miles per hour. And if the given values were 110.0 miles and 55.00 miles per hour, the correct answer would be different yet again. Mathematically, of course, all three answers are going to be the same. If you drive 110 miles at 55 miles per hour, the trip will take two hours. But scientifically, the correct answers to these three problems are different: 2.0 hours, 2.00 hours, and 2.000

hours, respectively. The difference between these cases is in the precision indicated by the given data, which are *measurements*. (Even though this is just a made-up problem in a book and not an actual measurement someone made in an experiment, the given data are still measurements. There is no way to talk about distances or speeds without talking about measurements, even if the measurements are only imaginary or hypothetical.)

So when you perform a calculation with physical quantities (measurements), you can't simply write down all the digits shown by your calculator. The precision inherent in the measurements used in a computation governs the precision in any result you might calculate from those measurements. And since the precision in a measurement is indicated by the number of significant digits, data and calculations must be written with the correct numbers of significant digits. To do this, you need to know how to count significant digits, and you must use the correct number of significant digits in all of your calculations and experimental data.

Correctly counting significant digits involves four different cases:

1. Rules for determining how many significant digits there are in a given measurement.
2. Rules for writing down the correct number of significant digits in a measurement you are making and recording.
3. Rules for computations you perform with measurements—multiplication and division.
4. Rules for computations you perform with measurements—addition and subtraction.

We will address each of these cases, in order.

#### Case 1

We begin with the rule for determining how many significant digits there are in a given measurement value. The rule is as follows:

- *The number of significant digits in a number is found by counting all the digits from left to right, beginning with the first nonzero digit on the left. When no decimal is present, trailing zeros are not considered significant.*

As examples of this rule, consider the following:

- 15,679      This value has five significant digits.
- 21.0005     This value has six significant digits.
- 37,000      This value has only two significant digits because when there is no decimal, trailing zeros are not significant. Notice that the word *significant* here is a reference to the precision of the measurement, which in this case is rounded to the nearest thousand. The zeros in this value are certainly *important*, but they are not *significant* in the context of precision.
- 0.0105      This value has three significant digits because we start counting digits with the first nonzero digit on the left.
- 0.001350    This value has four significant digits. Trailing zeros count when there is a decimal.

The significant digit rules enable us to tell the difference between two measurements such as 13.05 m and 13.0500 m. Again, these values are obviously equivalent *mathematically*. But they are different in what they tell us about the process of how the measurements were made—and science deals in measurements. The first measurement has four significant digits. The second measurement is more precise. It has six significant digits, and was made with a more precise instrument.

Now, just in case you are bothered by the zeros at the end of 37,000 that are not significant, here is one more way to think about significant digits that may help. The precision in a measurement depends on the instrument used to make the measurement. If we express the measurement in different units, this cannot change the precision of the value. A measurement of 37,000 grams is equivalent to 37 kilograms. Whether we express this value in grams or kilograms, it still has two significant digits.

### Case 2

The second case addresses the rules that apply when you are recording a measurement yourself, rather than reading a measurement someone else has made. When you take measurements yourself, as you do in laboratory experiments, you need to know the rules for which digits are significant in the reading you are taking on the measurement instrument. The rule for taking measurements depends on whether the instrument you are using is a digital instrument or an analog instrument. Here are the rules for these two possibilities:

- **Rule 1 for digital instruments** *For the digital instruments commonly found in high school or undergraduate science labs, assume all of the digits in the reading are significant except leading zeros.*
- **Rule 2 for analog instruments** *The significant digits in a measurement include all of the digits known with certainty, plus one digit at the end that must be estimated between the finest marks on the scale of your instrument.*

The first of these rules is illustrated in Figure 1.3. The reading on the left has leading zeros, which do not count as significant.



Figure 1.3. With digital instruments, all digits are significant except leading zeros. Thus, the numbers of significant digits in these readings are, from left to right, three, three, five, and five.

Thus, the first reading has three significant digits. The second reading also has three significant digits. The third reading has five significant digits. The fourth reading also has five significant digits because with a digital display, the only zeros that don't count are the leading zeros. Trailing zeros are significant with a digital instrument. However, when you write this measurement down, you must write it in a way that shows those zeros to be significant. The way to do this is by using scientific notation. Thus, the right-hand value in Figure 1.3 must be written as  $4.2000 \times 10^4$ .

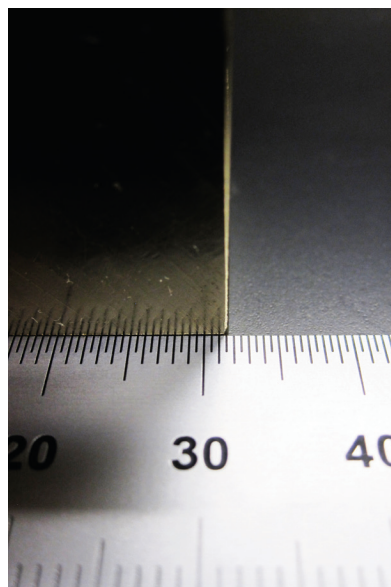


Figure 1.4. Reading the significant digits with a machinist's rule.

Dealing with digital instruments is actually more involved than the simple rule above implies, but the issues involved go beyond what we typically deal with in introductory or intermediate science classes. So, simply take your readings and assume that all the digits in the reading except leading zeros are significant.

Now let's look at some examples illustrating the rule for analog instruments. Figure 1.4 shows a machinist's rule being used to measure the length in millimeters (mm) of a brass block. We know the first two digits of the length with certainty; the block is clearly between 31 mm and 32 mm long. We have to estimate the third significant digit. The scale on the rule is marked in increments of 0.5 mm. Comparing the edge

of the block with these marks, I would estimate the next digit to be a 6, giving a measurement of 31.6 mm. Others might estimate the last digit to be 5 or 7; these small differences in the last digit are unavoidable because the last digit is estimated. Whatever you estimate the last digit to be, two digits of this measurement are known with certainty, the third digit is estimated, and the measurement has three significant digits.

The photograph in Figure 1.5 shows a liquid volume measurement in milliliters (mL) being made with a buret. The scale is marked in increments of 0.1 mL. This means we are to estimate to the nearest 0.01 mL. To one person, it may look like the bottom of the meniscus (the black curve) is just below 2.2 mL, so that person would call this measurement 2.21 mL. To someone else, it may seem that the bottom of the meniscus is right on 2.2, in which case that person would call the reading 2.20 mL. Either way, the reading has three significant digits and the last digit is estimated to be either 1 or 0.

As a third example, Figure 1.6 shows a liquid volume measurement being made in a graduated cylinder. The scale on the graduated cylinder is marked in increments of 1 mL. In the photo, the entire meniscus appears silvery in color with a black curve at the bottom. For the liquid shown in the figure, we know the first two digits of the volume measurement with certainty, because the reading at the bottom of the meniscus is clearly between 82 mL and 83 mL. We have to estimate the third digit, and I would estimate the black line to be at 40% of the distance between 82 and 83, giving a reading of 82.4 mL.

It is important for you to keep the significant digits rules in mind when you are taking measurements and entering data for your lab reports. The data in your lab journal and the values you use in your calculations and report should correctly reflect the use of the significant digits rules as they apply to the actual instruments you use to take your measurements.

### Case 3

The third and fourth cases of rules for significant digits apply to the calculations you perform with measurements. In Case 3 we will deal with multiplication and division. The main idea behind the rule for multiplying and dividing is that the precision you report in your result cannot be higher than the precision that was in the measurements to start with. The precision in a measurement depends on the instrument used to make the measurement, nothing else. Multiplying and dividing things cannot improve that precision, and thus your results can be no more precise than the measurements that went into the calculations. In fact, your result can be no more precise than the *least precise value* used in the calculation. The least precise value is, so to speak, the “weak link” in the chain, and a chain is no stronger than its weakest link.

Here are the two rules for using significant digits in calculations involving multiplication and division:

- **Rule 1** When multiplying or dividing, count the significant digits in each of the values you will use in a calculation, including any conversion factors you are using. (However,



Figure 1.5. Reading the significant digits on a buret.



Figure 1.6. Reading the significant digits on a graduated cylinder.

*note: conversion factors that are exact are not considered.) Determine how many significant digits there are in the least precise of all of these values. The result of your calculation must have this same number of significant digits.*

- **Rule 2** When performing a multi-step calculation, keep at least one extra digit during intermediate calculations, and round to the final number of significant digits you need at the very end. This practice will assure that small round-off errors don't accumulate during a multi-step calculation. This extra digit rule also applies to unit conversions performed as part of the computation.

#### Case 4

The fourth case of rules for significant digits also applies to the calculations you perform with measurements. In Case 4 we will deal with addition and subtraction.

The rule for addition and subtraction is completely different than the rule for multiplication and division. When performing addition, it is not the number of significant digits that governs the precision of the result. Instead, it is the *place value of the last digit that is farthest to the left in the numbers being added* that governs the precision of the result. This rule is quite wordy and is best illustrated by an example. Consider the following addition problem:

$$\begin{array}{r} 13.65 \\ 1.9017 \\ + 1,387.069 \\ \hline 1,402.62 \end{array}$$

Of the three values being added, 13.65 has digits out to the hundredths place, 1.9017 has digits out to the ten thousandths place, and 1,387.069 has digits out to the thousandths place. Looking at the final digits of these three, you can see that the final digit farthest to the left is the 5 in 13.65, which is in the hundredths place. This is the digit that governs the final digit of the result. There can be no digits to the right of the hundredths place in the result. The justification for this rule is that one of our measurements is precise only to the nearest hundredth, even though the other two are precise to the nearest thousandth or ten thousandth. We are going to add these values together, and one of them is precise only to the nearest hundredth. Thus, it makes no sense to have a result that is precise to a place more precise than that, so hundredths are the limit of the precision in the result.

Correctly performing addition problems in science (where nearly everything is a measurement) requires that you determine the place value governing the precision of your result, perform the addition, and then round the result. In the above example, the sum is 1,402.6207. Rounding this value to the hundredths place gives 1,402.62.

### 1.2.4 Random and Systematic Error

The two main types of error in experimental measurements are *random error* and *systematic error*. Random errors are caused by unknown and unpredictable fluctuations in the experimental setup. Examples of random error would be changes in the apparatus due to temperature fluctuations in the room, vibrations or wind that influence the measurement in a random fashion, or electronic noise that influences the readings in your instruments. When you calculate and discuss the uncertainty in your measurements, you are discussing the random error that caused your measurements to fluctuate randomly around the mean value.

Systematic errors are errors that bias the experimental results in one direction, and are usually caused by equipment defects, miscalibration of measurement instruments, or an experimenter who consistently misreads or misuses the instruments in the same way. Usually, when



discussing systematic error, we are talking about problems that could be eliminated by proper use, calibration, and operation of the equipment.

Your percent difference values (Section 1.2.6) can be influenced by factors you did not take into account in your predictions, and the result can be percent difference values that look like they include systematic error. A common example of this is physics experiments involving motion that do not take friction into account. If you make predictions in a mechanical system without taking friction into account, your results will all be biased in the same way relative to your measurements. This is *not* an “experimental error”; it is the result of using approximations in your theoretical modeling of the experiment. However, it is a contributor to your percent difference values and could play a role in the discussion of your results.

### 1.2.5 Standard Deviation

No doubt, you are already familiar with the statistical parameters *mean* and *median*. In the language of statistics, these parameters are *measures of center*—they indicate where the “middle” of a data set is. The *standard deviation* of a data set is another statistical parameter, one that indicates the amount of “spread” in a data set. There are two different calculations for the standard deviation: the population standard deviation ( $\sigma$ ) and the sample standard deviation ( $s$ ). The population standard deviation applies to data sets that include every member of a population; the sample standard deviation applies to data sets that contain a sample of values from a population, but not the entire population. For example, it is possible to know the age of every single student in a school, and such a data set constitutes a population. But since a data set of scientific measurements almost never contains every possible measurement value that could occur, sets of measurement data are usually samples and the sample standard deviation is the one most commonly used with scientific data.

When taking measurements or performing calculations from repeated trials of an experiment, the values of the measurements—if there are enough of them—often form a *Gaussian distribution* (also called a *normal distribution*). Figure 1.7 illustrates how measurements form a Gaussian distribution. Each circle in the figure represents a measurement. Most of the measurement values are close to the mean value, so the distribution peaks here; some measurements lie far above or below the mean, giving the distribution its “tails.”

Figure 1.8 shows two more typical Gaussian distributions, the second distribution more spread out than the first. The standard deviation,  $s$ , of a data set is a measure of how spread out the data are. Larger values of  $s$  mean wider spread; smaller values of  $s$  mean a narrower spread. The more accurate your experimental methods, and

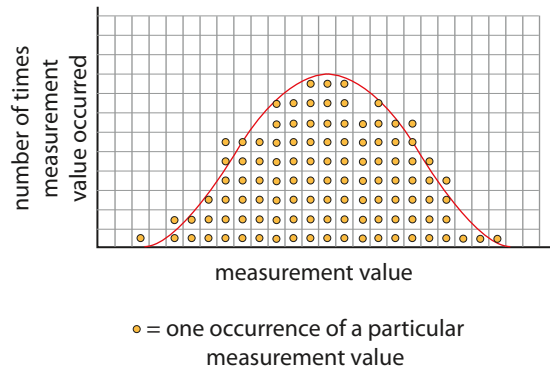


Figure 1.7. A Gaussian distribution forms from the data when a specific measurement is made repeatedly with an instrument precise enough to show the variation.

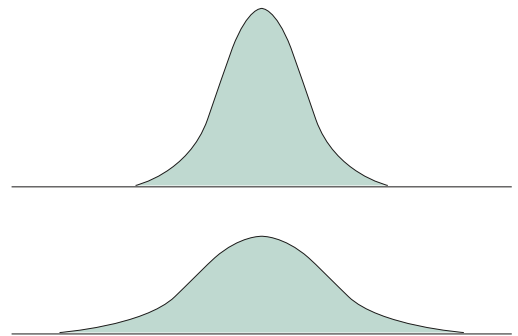


Figure 1.8. Gaussian distributions: narrow, with smaller  $s$  (top), and wide, with larger  $s$  (bottom).

the more precise your instruments, the narrower the spread in the data should be (all else being equal). Data that are very close together will have a very small value of  $s$ —exactly what you want. If all the data have exactly the same value, then  $s = 0$ .

To help you understand the standard deviation a bit more, Figure 1.9 indicates the relationship between the standard deviation of a data set and the shape of the data distribution (assuming a Gaussian distribution). The mean of the distribution is in the center, where the zero is on the horizontal scale. The scale is marked in increments of one standard deviation:  $1s$ ,  $2s$ , and so on. In a Gaussian data distribution, 68% of the data values will lie within one standard deviation of the mean, 95.4% of the data will lie within two standard deviations of the mean, and

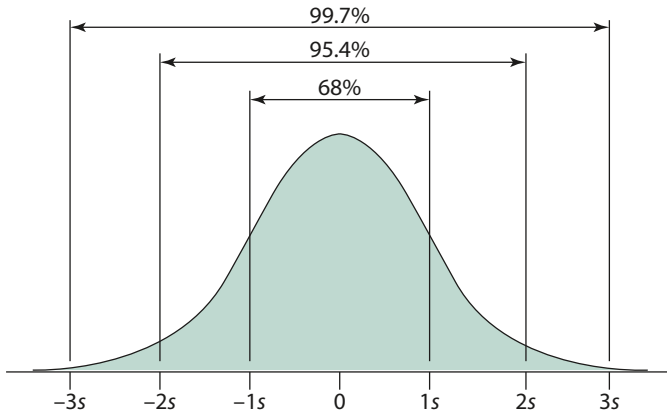


Figure 1.9. Relationship between the standard deviation and the area under the curve for a Gaussian distribution.

99.7% of the data will lie within three standard deviations of the mean. Note that the units for  $s$  are the same as the units of the data. If your data are in mL,  $s$  will also be in mL. If the mean of the data is 75.5 mL and  $s = 3.1$  mL, then 68% of the data lie between 72.4 mL and 78.6 mL. If  $s$  is only 1.1 mL, then 68% of the data lie between 74.4 mL and 76.6 mL—a narrower distribution with less *uncertainty* in the measurements.

The sample standard deviation is often used as a measure of uncertainty in a data set. As I wrote above, all measurements

contain error; that's just a fact of life. All measurements made with enough precision will show variation, and together the values will form a distribution, indicating that there is uncertainty as to the true value of the parameter being measured. Quoting the value of  $s$  for a set of data is a very common way of indicating the uncertainty in the data. All computer spreadsheet applications and graphing calculators can calculate the sample standard deviation. Just enter your data and look up how to run the calculation of the sample standard deviation on your device.

### 1.2.6 Calculating Percent Difference

The topic in this section applies primarily to the experiments you will perform as part of your study of physics. I am including the topic here because it addresses one of the common mathematical tools we use to analyze the data taken in an experiment and compare them to the predictions we would make based on our theoretical understanding of how nature works. (We will address the topic of theories in the next section.)

One of the conventional calculations in high school and college physics experiments is the so-called “experimental error.” Experimental error is typically defined as the difference between the predicted value (which comes from scientific theory) and the experimental value, expressed as a percentage of the predicted value, or

$$\text{experimental error} = \frac{|\text{predicted or accepted value} - \text{experimental value}|}{\text{predicted or accepted value}} \times 100\%$$

Although the term “experimental error” is widely used, it is in my view a poor choice of words. When there is a mismatch between theory and experiment, the experiment may not be the source of the error. Often, it is the theory that is found wanting. This is how science advances.

It is, of course, true that at the introductory and intermediate level students are not generally engaged in research that uncovers weaknesses in scientific theories. At this level, the difference between prediction and experimental result may well be due entirely to “experimental error” arising from experimental limitations or inaccuracies. However, I prefer that students develop scientific habits of mind, and in the real world of scientific research in physics and engineering, the measurements are as accurate as the experimenters know how to make them, and one does not know whether differences between mathematical prediction and experimental result are due to the mathematical model or error in the experiment.

I prefer to use the phrase *percent difference* to describe the value computed by the above equation. When quantitative results are compared to quantitative predictions or accepted values, students should compute the percent difference as

$$\text{percent difference} = \frac{|\text{predicted or accepted value} - \text{experimental value}|}{\text{predicted or accepted value}} \times 100\%$$

One more note. In the study of statistics, there is a calculation call the “percentage difference,” in which the difference between two values is divided by their average. To avoid potential future confusion, you should note the distinction between the calculation we are using here and the one arising in statistics.

## 1.3 Modeling Nature

### 1.3.1 *Science as Mental Model Building*

Students usually find the study of physics to be fascinating and rewarding—even though it is also very challenging. This is because physics is all about modeling the fundamental interactions of the matter and energy the world is composed of. The fact that humans can accurately model nature with mathematics is a wonderful consequence of God’s design: God made the world in such a way that it lends itself to mathematical characterization, and God made human beings with the mental ability to imagine mathematical structures. The world’s mathematical properties and our mathematical abilities fit together extremely well, and the result is that over the past 400 years, the accomplishments in the field of physics have been nothing short of stupendous.

Before we dive into modeling nature with mathematics, we need to pause and consider again how the modeling process works in science, and what kind of knowledge science provides for us. It is helpful to think of science as the process of building “mental models” of the natural world. These mental models are called *theories*. The information we use to build our mental models—scientific facts—comes from experiments, observations, and inferences from these.

The theoretical models developed by scientists are the basis for our entire understanding of how the natural world functions. Successful theories are those that account for the facts we know and lead to new hypotheses (predictions) that can be put to the test. It is helpful to think about the relationship between facts, theories, hypotheses, and experiments as illustrated in Figure 1.10. This diagram illustrates what I call the *Cycle of Scientific Enterprise*. You may have studied this diagram before in a previous class. It is important for every student to develop a correct understanding of the kind of knowledge scientific study provides for us. The *goal* of science is to uncover the truth about how nature works, but scientific theories are always works in progress. Even our best theories are provisional and subject to change. For this reason, science



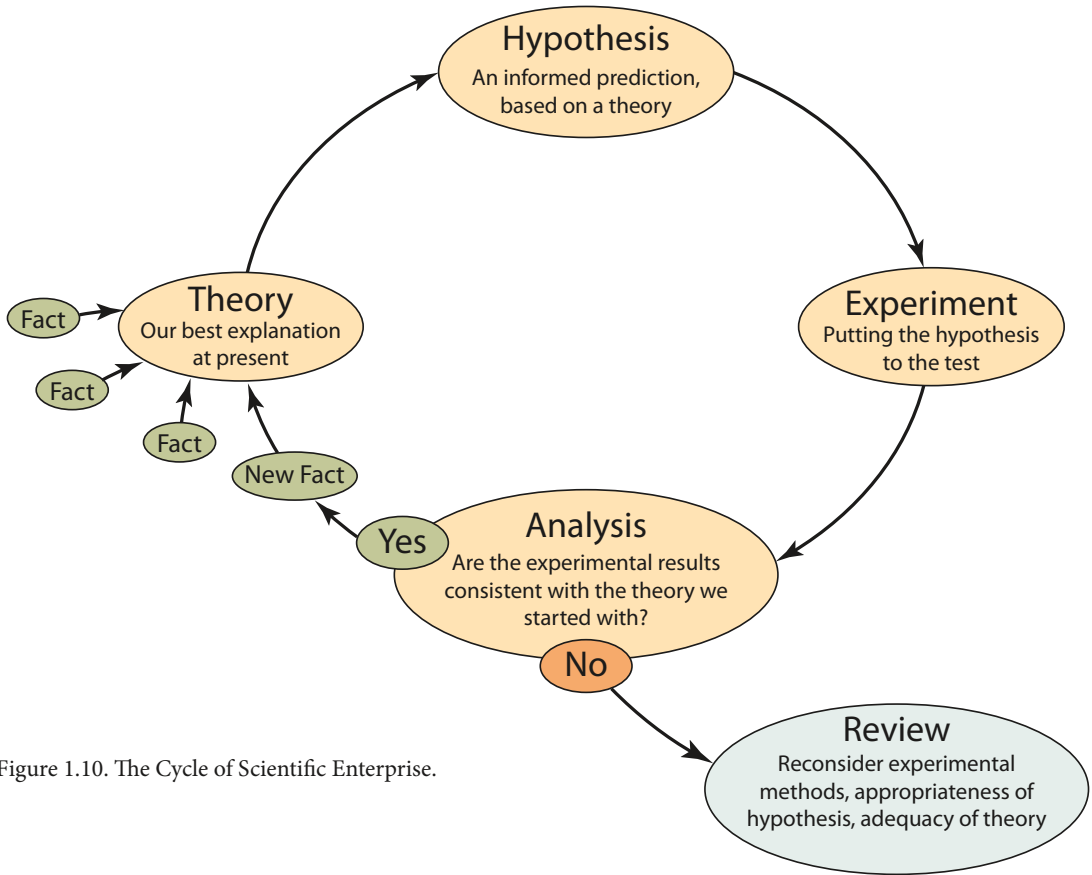


Figure 1.10. The Cycle of Scientific Enterprise.

is not in the business of making truth claims. It is in the business of modeling how nature works with theories based on research.

As our theories develop over time, our hope is that they get closer and closer to the truth, the amazing and profound truth about mysteries such as what protons and electrons are, why they have the properties they have, and how the two most successful theories of the 20th century—quantum mechanics and general relativity—can be reconciled with each other. But the truth about nature is always out in front of us somewhere, always outside of our grasp. To know the truth about nature we would have to understand nature as God understands it. We are nowhere close to that.

For this reason, all scientific knowledge—facts and theories—is regarded as *provisional*. Facts may be regarded as correct, and theories may be regarded as our best approximation to the reality of nature, but all such knowledge is subject to change, to being replaced by more accurate facts or theories.

### 1.3.2 Truth and Facts

Christians believe in truths that have been revealed to us, that are absolute and unchanging. Scientific facts, by their very nature, are not like this, so definitions for *truth* and for *scientific facts* must take this into account. First we will look at two ways that humans can know truth.

One way we know truth is when it is *evident* or *obvious* to us. For example, it is evident or obvious to you that you are awake right now while you are reading this text. Thus, it is correct to say that it is true you are awake. Likewise, you probably know if you have eaten a meal within

the past three days. If so, then it is obvious to you that if you said, “I have eaten a meal within the past three day” you would be speaking the truth.

Knowing obvious truths is clearly an important part of living as a human being in God’s world. God has made us so we can know truth about the world around us just by being here, seeing things, and remembering things. We rely on the stability of the world around us and our memories of it, because if it were otherwise we would probably all go crazy. The stability of the world and our memories are parts of the gift of creation God has given to us.

A second important way for us to know truth is for God to *reveal* it to us. Much truth is knowledge that is revealed to us by God, either by Special Revelation or by General Revelation. Special Revelation is the term theologians use to describe truths God teaches us in the Bible, his Holy Word. General Revelation refers to truths God teaches us through the world he made. Sometimes people describe Special and General Revelation as the two “books” of God’s revelation to us, the book of God’s *Word* (the Bible) and the book of God’s *Works* (nature).

Since truth is obvious, or revealed by God (either through his word or his works), this means that truth is not discovered the same way scientific facts are discovered. Truth is true for all people, all times and all places. Truth is permanent and never changes. Some examples of revealed truths are:

- Jesus is the divine Son of God (Matthew 16:16).
- All have sinned and fall short of what God requires (Romans 3:23).
- All people must die once and then face judgment (Hebrews 9:27).
- God is the creator of all that is (Colossians 1:16, Revelation 4:11).
- God loves us (John 3:16).

Each of these statements is true, and we know they are true because God has revealed them to us in his word. (The reasons for believing God’s word are important for all of us to know and understand, but that is a subject for a different course of study.) These truths are all unchanging, just like all truth. The distinction between truth and scientific facts, which I will describe in the next section, is crucial. As believers, we embrace the absolute truths we find in Scripture. Facts can change; truth does not. If we confuse these terms we may have a hard time discussing and defending the faith and distinguishing between the precious truths we know absolutely, such as that Jesus is our redeemer and rose from the dead, and things that may turn out not to be correct after all.

### 1.3.3 Facts, Theories, Hypotheses, and Experiments

To expound on the Cycle of Scientific Enterprise a bit further, the following are some definitions to keep in mind as you consider the mathematical models we develop in coming chapters.

#### Scientific Facts

A fact is a proposition based on a large amount of scientific data that is correct as far as we know. Facts are discovered by experiment, observation, and inferences from experiments and observations. Facts can and do change as new scientific knowledge—new data—is acquired. Since facts are always subject to change, careful scientists will usually avoid terms such as *true* or *proven* to describe facts. Instead, we say a fact is correct as far as we know.

Scientific facts are just a small step away from raw data. To talk about what facts mean, we must relate them together in a consistent explanatory framework. This is where theories come in.

#### Theories

A theory is a mental model that accounts for the data (facts) in a certain field of research, and attempts to relate them, interpret them, and explain them. Scientific theories are successful if they repeatedly allow scientists to form new hypotheses that can be confirmed by experiment. *Successful theories are the glory and goal of science.* Nevertheless, theories, like facts,

are provisional and subject to change. Indeed, theories are almost constantly evolving as research continues. And as with facts, when referring to theories, we avoid terms like *true* or *proven*. Instead, we speak in terms of how successful theories have been in generating hypotheses that are confirmed by experiments, that is, how accurately predictions derived from the theory match the results of experiments. A widely accepted scientific theory should be understood as our best explanation at present—our best model of how nature works.

It is important to realize that there is no scientific knowledge that is not theoretical. That is, data or facts by themselves don't tell us anything apart from the theories we have to account for and explain the facts. For example, it is a scientific fact that radium is radioactive; a lump of it glows constantly from the radiation it emits. But so what? What does this bare fact mean and what is its significance? What is radiation and what causes it? Our present theoretical model associates this type of radiation with the atomic nucleus and the process of nuclear decay. Some nuclear structures are less stable than others, and larger atoms with unstable nuclei typically exhibit nuclear decay until the nucleus reaches the stable structure of the nucleus of lead, element 82. Each different radioactive element decays at a rate that is very predictable using a statistical mathematical model. With this theoretical basis, we understand the behavior of the atomic nucleus. The fact of nuclear decay is explained and understood in terms of nuclear theory.

It is also important to note that scientific theories need to be testable. A theory that does not lead to testable hypotheses has no chance of gaining credibility and remains at the level of conjecture. For a theory to be well established, it must lead to hypotheses that can be put to the test.

### Hypotheses

A hypothesis<sup>1</sup> is an informed prediction about what will happen in certain circumstances. Every hypothesis is based on a particular theory. It is hypotheses that are tested and thereby confirmed or disconfirmed by scientific experiments. To form an experimental hypothesis, the scientist must understand the subject at issue according to a theoretical framework of some kind. This theoretical framework will determine the hypotheses scientists form and test.

For an example, we can continue with the theory of nuclear decay. We have found that all the elements discovered so far with atomic numbers  $Z = 84$  and higher are radioactive. Our theory of nuclear stability accounts for this. An obvious hypothesis we could form regarding other heavy elements not yet discovered—elements with atomic numbers  $Z = 119$  and higher—is that when we are able to identify such an element, we will expect atoms of the element to decay due to the instability of their nuclei.

### Experiments

An experiment is a test designed to confirm or disconfirm a particular hypothesis. If a hypothesis is confirmed through experiment, and if other scientists are able to validate the confirmation by replicating the experiment, then the new facts gained from the experimental results become additional support for the theory the hypothesis came from.

Continuing again with the theory of nuclear decay as an example, experiments in recent decades have been conducted repeatedly to identify the elements in the last period of the Periodic Table of the Elements. Since plutonium,  $Z = 94$ , is the heaviest naturally occurring element, scientists have expected that any element discovered with an atomic number greater than 94 will exhibit nuclear decay. And so far, experiments have repeatedly demonstrated this to be the case.

Sometimes, an experiment or series of experiments disconfirms a hypothesis. When this occurs, it is not correct to say that the theory that led to the hypothesis is immediately disproved. In fact, there are many factors that could lead to a disconfirmation. This is why the Cycle of Scientific Enterprise includes the steps called *Analysis* and *Review*. The Review process essentially works in reverse around the Cycle of Scientific Enterprise to discover why the hypothesis

---

1 A generation ago, correct usage required us to write *an hypothesis*, and some writers continue this usage today. I now use the more common contemporary usage *a hypothesis*.

was not confirmed. The first step in the review process is to review the experiment: protocols, instruments, data collection methods, apparatus, lurking variables, and a host of other factors that could have led to the negative result. If the experimental data appear to be valid, the review process turns next to the hypothesis. Contemporary theories are mathematically complex and it is possible that the hypothesis was not properly formed from the theory. If the hypothesis checks out, the final step is to examine the theory itself. A disconfirmed hypothesis means the theory did not deliver a correct prediction, and such a failure calls the theory's integrity into question. No theory is perfect and no theory explains all relevant data. A failed prediction indicates that there is more to the picture than scientists know about. A long series of failed predictions wears down a theory's credibility, setting the stage for eventual replacement by a new theory that explains all the data more adequately than its predecessor. However, the complexity of scientific theories these days is such that producing a new theory can be very challenging.

Physics is a subject loaded with facts and heavily based on theories—models—that we know are incomplete descriptions of nature. That is why the research continues, as our models (hopefully) get nearer and nearer to the truth.

## 1.4 Vector Methods

### 1.4.1 Scalars and Vectors

When we model nature mathematically, we describe the quantities we seek to measure with variables and we develop equations that reflect our understanding of the relationships between the variables that are present in nature. One of the most fundamental aspects of describing quantities with variables is that simple quantities in nature may always be classified as one of two distinct types—*scalars* and *vectors*.<sup>2</sup> Representative examples of scalar and vector quantities are listed in Table 1.6. It is likely that the term *displacement* is unfamiliar. A displacement is a directed distance, or a distance along a certain direction. The discussion below will further clarify the meaning of this term.

A scalar quantity can be expressed with a single value that indicates the size or amount of the quantity. Vector quantities cannot be expressed in terms of a single value. Vector quantities are inherently directional in character, and to express the value of a vector quantity both the *magnitude* and *direction* of the quantity must be stated.

For example, the temperature at a certain spot in a room can be expressed with a single value, such as 21°C, and thus temperature is a scalar quantity. By contrast, forces are directional and are thus vectors, so to describe a force completely the direction of the force must be stated along with its magnitude. The gravitational attraction of the earth on an object is an example of a force, and regardless where the object is, the direction of the force on the object is down (that is, toward the center of the earth). As another example, the force on a surface due to the pressure of a gas or liquid is perpendicular to the surface.

Two of the common vector quantities in physics, displacement and velocity, have common terms that refer to the magnitude of the vector (rather than both the magnitude and direction).

Scalar Quantities	Vector Quantities
temperature	velocity
mass	acceleration
energy	force
pressure	displacement
density	momentum

Table 1.6. Representative scalar and vector quantities.

<sup>2</sup> There is a third type—*tensors*—but tensor mathematics is very complex and does not typically appear in introductory or intermediate physics courses.

The magnitude of a displacement vector is called *distance* and the magnitude of a velocity vector is called *speed*. Since *distance* and *speed* refer specifically to the magnitudes of vectors, they refer to scalar quantities. If you say that a man walked 5 km, you have stated the distance the man walked—a scalar quantity. If you say that a man walked 5 km due northeast, you have stated the displacement the man underwent—a vector quantity. We will address conventions for specifying a vector's direction in a moment, but I will just note that there are a few common types of problems in physics that use geographical references to specify the direction of a vector, as I just did in this illustration.

Likewise, the term *speed* is a scalar quantity that refers to the magnitude of a velocity, a vector quantity. Thus, someone can refer to the speed of an electron as 276 m/s. But to state the velocity of the electron requires also stating its direction, which requires an angular reference of some kind. For solving problems in physics, the angular reference is often handled by representing the vector in question, the velocity of an electron in this case, as an arrow on a coordinate system. The magnitude of the vector is represented by the length of the arrow, and the direction of the vector is represented by the angle the arrow makes relative to the positive  $x$ -axis (or horizontal axis) in the coordinate system, as illustrated in Figure 1.11. As in trigonometry, positive angles are measured counterclockwise from the horizontal axis. With this in mind, one way of stating the velocity of the electron in the figure would be to use a symbol for the angle: 276 m/s,  $\triangle = 42^\circ$ . Another way to state the velocity is to write it out: 276 m/s at an angle of  $42^\circ$  relative to the positive  $x$ -axis. A third and very common notation is to use variables—an italicized letter for the magnitude and an italicized Greek letter for the angle, as in  $v = 276 \text{ m/s}$ ,  $\theta_v = 42^\circ$ .

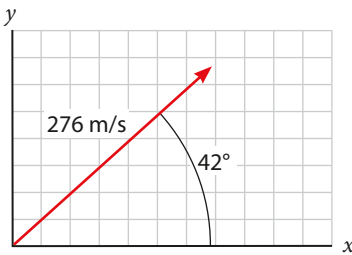


Figure 1.11. Specifying the direction of a vector quantity.

You are not necessarily required to use the positive  $x$ -axis for specifying angles, and there will be times when it is more convenient to use a different reference. Any convenient reference (such as the positive  $y$ -axis) may be used, so long as the reference is specified. As a convention in physics, directions are usually specified as relative to the positive  $x$ -axis. In the absence of any other specification, this is assumed.

The conventions for the notation used to denote vector quantities are summarized in Table 1.7. In print, different fonts are used to indicate scalar and vector quantities. An italic font is typically used for scalar quantities; a bold font is standard for vector quantities. In a text like this one, I indicate a distance of 5 km by typing  $d = 5 \text{ km}$ . To indicate the displacement of 5 km NE, I type  $\mathbf{d} = 5 \text{ km NE}$ . The electron speed would be printed as  $v = 276 \text{ m/s}$ . Its velocity could be printed with an angle symbol as  $\mathbf{v} = 276 \text{ m/s}$ ,  $\triangle = 42^\circ$ . However, not everyone recognizes the angle symbol, so it is more common (and formal) to use a variable for both the magnitude and direction, like this:  $v = 276 \text{ m/s}$ ,  $\theta = 42^\circ$ . If necessary, subscripts are used on the angle variables to make it clear which vector the angle is associated with:  $v = 276 \text{ m/s}$ ,  $\theta_v = 42^\circ$ . Another standard

notation is to use the absolute value symbol to indicate a vector's magnitude, and thus write  $|\mathbf{v}| = 276 \text{ m/s}$ ,  $\theta = 42^\circ$ .

Obviously, it is difficult to use italic and bold fonts when you are writing by hand on your exercises and exams. In that case, the convention is to place a little right-pointing arrow over a variable if you wish to indicate a vector, as shown in the table. Occasionally, printed mate-

	In Print	By Hand
Scalar	$d, v$	$d, v$
Vector	$\mathbf{d}, \mathbf{v}$	$\vec{d}, \vec{v}$
Magnitude of Vector	$d, v,  \mathbf{d} ,  \mathbf{v} $	$d, v,  \vec{d} ,  \vec{v} $
Angle of Vector	$\theta, \triangle$	$\theta, \triangle$

Table 1.7. Notation conventions for scalar and vector quantities.

rials will use both bold print and an arrow over variable, such as  $\vec{v}$ . When writing by hand, the most common way to indicate the magnitude of a vector is to use the absolute value symbol with the variable inside with an arrow over it:  $|\vec{d}|$ .

### 1.4.2 How to Learn Vector Addition

You already know how to solve problems using scalar quantities. You simply add, subtract, multiply, divide, and so on. When we add numbers this way we are adding *algebraically*. However, solving problems involving vector quantities requires using *vector arithmetic*. When we add two vector quantities together we must add them *vectorially*. Since a large percentage of problems in physics involve vectors, it is essential that you master the rules of vector arithmetic immediately. I encourage you to study this section carefully and work the exercises until you are certain you can execute vector calculations just as reliably and efficiently as you can execute scalar calculations.

When solving vector problems in physics, graphical aids are universally used to set the problem up and put together a solution strategy. You will see this over and over in this text and you will need to draw similar graphical depictions to aid you in your solutions. As I explained above, vector quantities are represented graphically as arrows on a coordinate system. The length of the arrow represents the magnitude of the vector, and the direction of the vector is indicated by the angle it makes with a reference line (typically, the positive  $x$ -axis).

We will first consider vector addition (Sections 1.4.3 through 1.4.5). There are two ways to execute a calculation involving vector addition. One is to perform the calculation graphically, using a rule and protractor. This is a time-consuming method and the accuracy of your results will depend entirely on the accuracy of your drafting technique. The second method is to use trigonometry to work out an analytical solution. This method will deliver an accurate answer (as long as you do it correctly) and is much faster.

You might be thinking that we should dispense with the graphical approach and get straight to the trigonometric approach. However, decades of experience teaching high school physics has convinced me that you should learn the graphical method first and solve a few problems with it so that you learn to visualize what you are doing. After that, you can learn the trigonometric approach and use it exclusively from that point onwards. I have seen many occasions when students tried to jump into the trigonometric methods of vector arithmetic without learning the graphical approach first. Most of the time this results in students having very little understanding of what they are doing, and this leads to disaster when we begin solving problems using vector methods. The moral of this story is that you need to spend a day with your rule and protractor getting the hang of this, even though it may seem like you're back in Geometry class. A couple of hours of practice will pay off and help ensure that you quickly and effectively learn the trigonometric methods to follow.

After we address vector addition in the next three sections, we will address vector multiplication in Sections 1.4.6 and 1.4.7. You will not need to know the methods presented in Section 1.4.7 until we get to the chapters on Static Equilibrium (Chapter 4) and Energy (Chapter 5). But I include the material in this chapter with the other material on vector arithmetic to make it easy to find later. For now, study Sections 1.4.3 through 1.4.5 carefully and do the problems in the exercises. Then proceed to Chapter 2. When the time comes that you need to know about vector multiplication, I will send you back to Section 1.4.7 to learn about it.

### 1.4.3 Vector Addition—the Graphical Method

Two vectors, **A** and **B**, are shown in Figure 1.12. Each has a tip and a tail, as indicated. The angle a vector makes is measured at the tail of the vector, as if the tail of the vector were placed at the origin of the coordinate system. The lengths of the vectors are drawn proportionally, us-



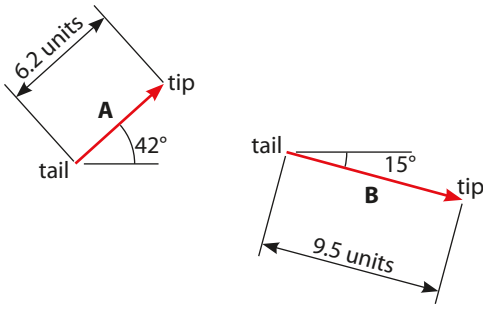


Figure 1.12. Graphical representations of two vectors, **A** and **B**.

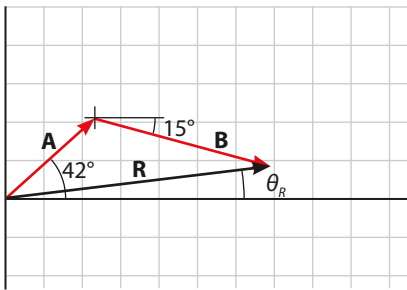


Figure 1.13. Graphical addition of vectors **A** and **B** to produce resultant vector, **R**.

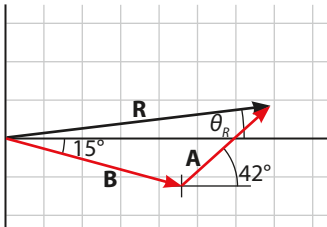


Figure 1.14. Adding vector **A** to vector **B** produces the same resultant as adding **B** to **A**.

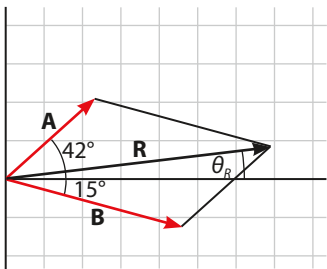


Figure 1.15. When both vectors **A** and **B** are drawn at the origin, they form adjacent sides of a parallelogram. The diagonal of the parallelogram is the resultant vector, **R**.

ing an appropriate scale, such as 1/4 inch = 1 unit, 1 inch = 10 units, etc. In the figure, vector **A** has a magnitude of 6.2 units, and is at an angle of  $42^\circ$  relative to the positive  $x$ -axis. (Just imagine the  $x$ -axis beginning at the vector's tail and pointing to the right. It is not necessary actually to draw an axis.) Vector **B** has a magnitude of 9.5 units, and is at an angle of  $-15^\circ$  relative to the positive  $x$ -axis. Note that in the diagram, the angle of **B** is labeled with a positive value. The geometric value of the angle between those two line segments is  $15^\circ$ . However, the *direction* of the vector must be stated as  $-15^\circ$ , because the  $15^\circ$  angle the vector makes with the

positive  $x$ -axis is in the clockwise direction, not the counterclockwise direction.

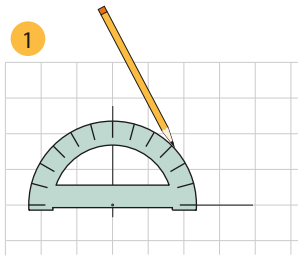
As with scalar addition, vector addition is commutative—several vectors may be added in any order and the result will be the same. The result of a vector addition is called the *resultant vector*, or simply the *resultant*. The resultant is a new vector, with its own magnitude and direction.

Graphical vector addition is illustrated in Figure 1.13. To add vectors **A** and **B**, place the tail of the second vector at the tip of the first vector. (Again, the operation is commutative; it doesn't matter which is first or second.) Preserve the vectors' lengths and angles while doing this. The resultant vector, **R**, is the vector whose tail is at the tail of the first vector and whose tip is at the tip of the last vector. In other words, draw **R** so that it points from the tail of the first vector to the tip of the last vector. Then measure the length of **R** with a rule to get the magnitude of **R**, denoted by  $|\mathbf{R}|$  or  $R$ , and use a protractor to measure the angle **R** makes with the positive  $x$ -axis, denoted by  $\theta_R$ . Drawing these vectors and making the measurements indicates that  $|\mathbf{R}| = 13.9$  units, and  $\theta_R = 7.0^\circ$ .

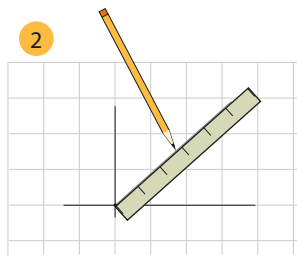
In Figure 1.14, the same two vectors are depicted but the order of the addition is reversed—vector **A** is added to **B**, instead of **B** being added to **A**. This diagram demonstrates the commutativity of vector addition, since the resultant **R** is the same vector as before.

In Figures 1.13 and 1.14, the vectors are added by drawing them tip to tail. However, Figure 1.15 demonstrates one additional feature of vector addition. If both vectors are drawn with their tails at the origin, they form adjacent sides of a parallelogram. Completing the other two sides, you can see that the resultant **R** of adding **A** + **B** is the diagonal of the parallelogram. Be careful here though: drawing a vector along the other diagonal, from the tip of **A** to the tip of **B**, would represent an *entirely different* addition problem: **A** + **R** = **B**. This is not the same as **A** + **B** = **R**. Take care to avoid this confusion.

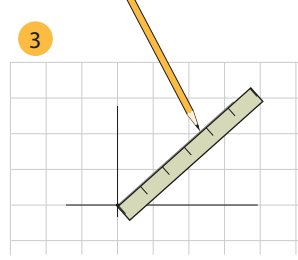
When adding vectors graphically, draw the vectors with a sharp pencil on graph paper, using as large a scale as possible. The lines on the graph paper will assist you in orienting your protractor and



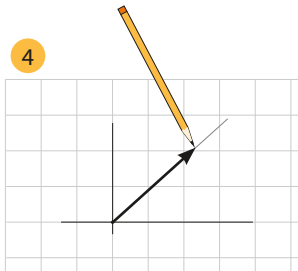
1 Set up axes and mark the angle of the first vector



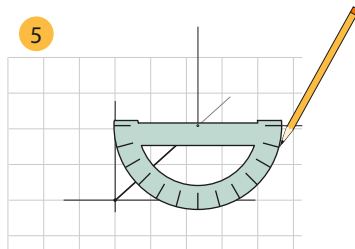
2 Draw a light line along where the vector will go



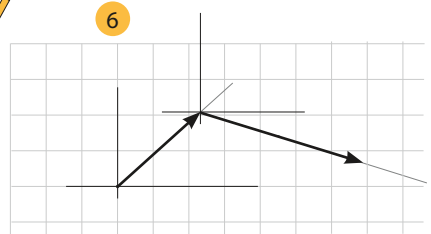
3 Mark the length of the vector on the line



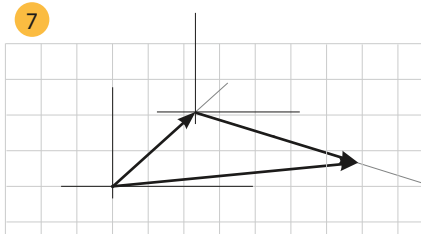
4 Draw the vector



5 Set up a new set of axes at the tip of the first vector, and mark the angle of the second vector



6 Measure and draw in the second vector as before



7 Draw the resultant

Figure 1.16. Drafting techniques for performing graphical vector addition.

a large scale will reduce the error caused by inaccuracies in your drawing. Additional drafting techniques for performing graphical vector addition are shown in Figure 1.16.

▼ Example 1.1

A girl is in a large flat meadow mapping out a playing field for a new game. From her starting location, she walks 21 meters north, then 27 meters southeast, then 11 meters northeast. a) What is the total distance the girl walks? b) Use graphical vector addition methods to determine the girl's displacement,  $\mathbf{d}$ , from the starting location after these maneuvers have been completed. c) At the end, how far is the girl from her starting point?

a) The total distance walked is simply the sum of the distances walked:

$$21 \text{ m} + 27 \text{ m} + 11 \text{ m} = 59 \text{ m}$$

b) The resultant vector from adding together the three displacement vectors gives us the girl's displacement at the end. Working this problem on regular 8.5 in  $\times$  11 in graph paper, a convenient scale to use would be 0.5 cm = 1 m. The three walking distances then become:



$$21 \text{ m} \cdot 0.5 \text{ cm} = 10.5 \text{ cm}$$

$$27 \text{ m} \cdot 0.5 \text{ cm} = 13.5 \text{ cm}$$

$$11 \text{ m} \cdot 0.5 \text{ cm} = 5.5 \text{ cm}$$

The vector addition is illustrated in Figure 1.17. We will let north be at the top of the page. Beginning at an origin placed toward the left side of the page, we first draw a vector with a length of 10.5 cm straight up. From the tip of this vector, we draw a second vector with a length of 13.5 cm at an angle of  $-45^\circ$ . From the tip of the second vector we draw a third vector with a length of 5.5 cm at an angle of  $45^\circ$ .

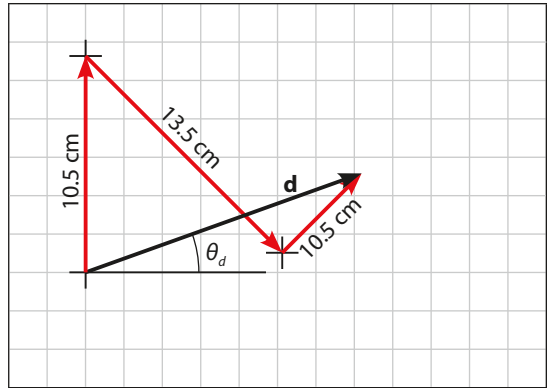


Figure 1.17. Three displacements and the resultant displacement  $\mathbf{d}$  for Example 1.1.

Drawing this problem myself as accurately as possible, I measured the resultant (the black vector in the figure) to be 14.25 cm long at an angle of  $20.0^\circ$ . Converting the scaled length back into meters:

$$14.25 \text{ cm} \cdot \frac{1 \text{ m}}{0.5 \text{ cm}} = 28.5 \text{ m}$$

This gives  $|\mathbf{d}| = 28.5 \text{ m}$  and  $\theta_d = 20.0^\circ$ . Rounding to two significant digits, we have  $|\mathbf{d}| = 29 \text{ m}$  and  $\theta_d = 2.0 \times 10^1$  degrees.

c) The distance from the starting point is the magnitude of the displacement vector. Thus,

$$d = 29 \text{ m}.$$



#### 1.4.4 Additional Details About the Direction of a Vector

There are two additional details to add regarding the way we represent the direction of a vector. First, any vector can be represented with either a positive or negative angle, as illustrated by the force vector in Figure 1.18. In other words,

$$35 \text{ N}, \triangle 102^\circ = 35 \text{ N}, \triangle -258^\circ$$

These are simply two different ways of representing the same vector. However, in practice, angles greater than  $180^\circ$  are used infrequently except when specifying a direction with a compass heading, also called *bearing*, illustrated in Figure 1.19. A bearing uses the  $360^\circ$  compass, with due north at  $0^\circ$ . Angles increase from zero by going clockwise from due north. (Thus, due east is a bearing of  $90^\circ$ .) The use of bearings for specifying angles comes up frequently in problems involving the navigation of ships and aircraft.

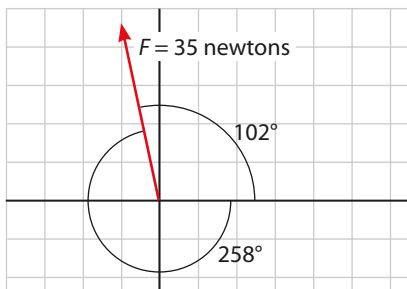


Figure 1.18. Vector  $\mathbf{F}$  can be represented equivalently as 35 N at an angle of  $102^\circ$  or as 35 N at an angle of  $-258^\circ$ .

The second point is quite important and comes up constantly. By convention, *the magnitude of a vector is always positive*. However, when solving vector addition problems using the trigonometric method (coming up next), we rou-

tinely encounter vectors or vector “components” (a term explained in the next section) that point in the direction of the positive  $x$ -axis, the negative  $x$ -axis, the positive  $y$ -axis, or the negative  $y$ -axis. We designate these vectors or vector components as positive if they point in the direction of the positive  $x$ - or  $y$ -axis; we designate them as negative if they point in the direction of the negative  $x$ - or  $y$ -axis. The important point to note here is that a negative sign in front of the magnitude of a vector quantity does not mean the magnitude is negative. The negative sign is understood to denote the direction of the vector. The use of a negative sign to indicate that a vector points in the direction of the negative  $x$ - or  $y$ -axis occurs frequently once we begin using trigonometry to handle our vector arithmetic. This is our next topic.

### 1.4.5 Vector Addition—the Trigonometric Method

You now know that any pair of vectors can be added together to produce a resultant vector. Of course, this implies that a given vector can be represented as the sum of an infinite number of different vector pairs. In particular, any vector can be represented as the sum of two vectors that are parallel to the  $x$ - and  $y$ -axes.

Going back to the electron we considered in Section 1.4.1, Figure 1.20 shows the electron’s velocity vector  $\mathbf{v}$  (in red) with its magnitude of  $v = 276$  m/s at an angle of  $\theta_v = 42.0^\circ$ . (I added a third significant digit to the angle for greater precision.) Also shown in the diagram are two other vectors parallel to the  $x$ - and  $y$ -axes labeled  $\mathbf{v}_x$  and  $\mathbf{v}_y$ . As you can see, the velocity vector is the resultant of the vector sum of  $\mathbf{v}_x$  and  $\mathbf{v}_y$ . Together, these three vectors form a right triangle. Simple right-triangle trigonometry allows us to calculate the magnitudes  $v_x$  and  $v_y$ :

$$v_x = 276 \frac{\text{m}}{\text{s}} \cdot \cos 42.0^\circ = 205 \frac{\text{m}}{\text{s}}$$

$$v_y = 276 \frac{\text{m}}{\text{s}} \cdot \sin 42.0^\circ = 185 \frac{\text{m}}{\text{s}}$$

The two vectors parallel to the  $x$ - and  $y$ -axes with magnitudes  $v_x$  and  $v_y$  are called the  $x$ - and  $y$ -components or *horizontal and vertical components* or *rectangular components* of  $\mathbf{v}$ . The great thing about these components is that they add up (vectorially) to  $\mathbf{v}$ . We can use this fact to develop a nice analytical method for calculating a vector sum. We do it by first determining the  $x$ - and  $y$ -components of the vectors being added, and then using these to determine the  $x$ - and  $y$ -components of the resultant. Here is the rule for obtaining the components of the resultant:

- *The  $x$ -component of the resultant of a vector sum is equal to the sum of the  $x$ -components of the vectors being added. Likewise, the  $y$ -component of the resultant of a vector sum is equal to the sum of the  $y$ -components of the vectors being added.*

This rule is very handy because adding up components is done algebraically—you just add them up, treating components that point up or to the right as positive and those that point down or to the left as negative. Once the components of the resultant have been determined, the magnitude and direction of the resultant can be determined using two additional relations from right-triangle trigonometry. Referring again to Figure 1.20, you can see that the following equations hold:

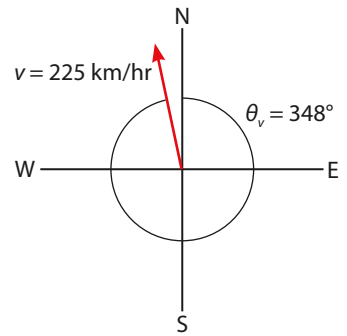


Figure 1.19. Velocity vector  $\mathbf{v}$  shown as a bearing, where the angle is measured clockwise from north with due north at  $0^\circ$ .

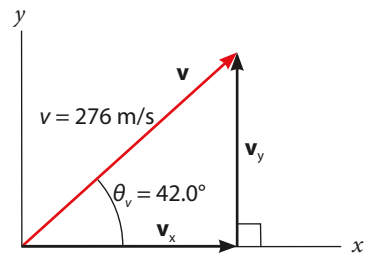


Figure 1.20. The  $x$ - and  $y$ -components of velocity vector  $\mathbf{v}$ .

$$|\mathbf{v}| = v = \sqrt{v_x^2 + v_y^2}$$

$$\theta_v = \tan^{-1} \frac{v_y}{v_x}$$

To illustrate this method, we will go back to our first graphical vector problem.

▼ Example 1.2

Determine the resultant,  $\mathbf{R}$ , of the sum of the following two vectors:

vector  $\mathbf{A}$ :  $A = 6.2$  units,  $\theta_A = 42^\circ$

vector  $\mathbf{B}$ :  $B = 9.5$  units,  $\theta_B = -15^\circ$

First, we determine the  $x$ - and  $y$ -components of  $\mathbf{A}$  and  $\mathbf{B}$ :

$$A_x = A \cos \theta_A = 6.2 \cos 42^\circ = 4.61 \text{ units}$$

$$B_x = B \cos \theta_B = 9.5 \cos(-15^\circ) = 9.18 \text{ units}$$

$$A_y = A \sin \theta_A = 6.2 \sin 42^\circ = 4.15 \text{ units}$$

$$B_y = B \sin \theta_B = 9.5 \sin(-15^\circ) = -2.46 \text{ units}$$

Notice that according to our standard practice for dealing with significant figures, I have kept an extra digit in these intermediate results. Now, the  $x$ -component of  $\mathbf{R}$ ,  $R_x$ , is the sum of  $A_x$  and  $B_x$ :

$$R_x = A_x + B_x = 4.61 \text{ units} + 9.18 \text{ units} = 13.79 \text{ units}$$

Likewise, the  $y$ -component of  $\mathbf{R}$ ,  $R_y$ , is the sum of  $A_y$  and  $B_y$ :

$$R_y = A_y + B_y = 4.15 \text{ units} - 2.46 \text{ units} = 1.69 \text{ units}$$

With the components of  $\mathbf{R}$ , we can determine its magnitude and direction,  $R$  and  $\theta_R$ :

$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(13.79 \text{ units})^2 + (1.69 \text{ units})^2} = 13.9 \text{ units}$$

$$\theta_R = \tan^{-1} \frac{R_y}{R_x} = \tan^{-1} \frac{1.69}{13.79} = 7.0^\circ$$

Comparing these results to the graphical results we obtained previously, we see that the values agree. But accuracy is difficult when measuring angles with a protractor; generally, it is easier to obtain accurate values with the trigonometric method.

Finally, following the significant digits in these calculations requires us to get quite pedantic. The angle is not difficult: we began with values containing two digits of precision, so our angle has two significant digits. The magnitude of  $\mathbf{R}$  is more complicated. Each of the squared values in the radical has an extra digit. Keeping only one extra digit at every step, here's what happens: Squaring 13.79 and 1.69 gives 190.2 and 2.86. Adding these drops a decimal, giving 193.1. The square root is 13.90, which still has one extra digit. Rounding gives 13.9, one more digit of precision than we started with. This often happens when using the addition rule.



As an additional example, we will rework Example 1.1 so we can compare our graphical result to the more accurate analytical result.

▼ Example 1.3

A girl is in a large flat meadow mapping out a playing field for a new game. From her starting location, she walks 21 meters north, then 27 meters southeast, then 11 meters northeast. a) What is the total distance the girl walks? b) Use trigonometric vector addition methods to determine the girl's displacement from the starting location after these maneuvers have been completed. c) At the end, how far is the girl from her starting point?

a) The total distance walked is still the sum of the distances walked:

$$21 \text{ m} + 27 \text{ m} + 11 \text{ m} = 59 \text{ m}$$

b) We will label the three displacements of the girl's walk as vectors  $\mathbf{d}_1$ ,  $\mathbf{d}_2$ , and  $\mathbf{d}_3$ . These three vectors are:

$$\mathbf{d}_1: d_1 = 21 \text{ m}, \theta_1 = 90^\circ$$

$$\mathbf{d}_2: d_2 = 27 \text{ m}, \theta_2 = -45^\circ$$

$$\mathbf{d}_3: d_3 = 11 \text{ m}, \theta_3 = 45^\circ$$

These vectors, along with the resultant we are seeking and all their components, are shown in Figure 1.21. As you can see, the diagram makes it apparent that the  $x$ -component magnitudes add up to  $R_x$  and the  $y$ -component magnitudes add up to  $R_y$ .

We begin by calculating the  $x$ - and  $y$ -components for each displacement vector. Notice that since  $\mathbf{d}_1$  is vertical, it has no horizontal component (that is,  $d_{1x} = 0$ ).

$$d_{1x} = d_1 \cos \theta_1 = 21 \text{ m} \cdot \cos 90^\circ = 0$$

$$d_{2x} = d_2 \cos \theta_2 = 27 \text{ m} \cdot \cos (-45^\circ) = 19.1 \text{ m}$$

$$d_{3x} = d_3 \cos \theta_3 = 11 \text{ m} \cdot \cos 45^\circ = 7.78 \text{ m}$$

The magnitude of the  $x$ -component,  $R_x$ , of the resultant,  $\mathbf{R}$ , is the sum of these  $x$ -components.

$$R_x = 0 \text{ m} + 19.1 \text{ m} + 7.78 \text{ m} = 26.9 \text{ m}$$

Now we repeat the calculations for the  $y$ -components. Notice that since  $\mathbf{d}_1$  is vertical, its vertical component is equal to itself.

$$d_{1y} = d_1 \sin \theta_1 = 21 \text{ m} \cdot \sin 90^\circ = 21.0 \text{ m}$$

$$d_{2y} = d_2 \sin \theta_2 = 27 \text{ m} \cdot \sin (-45^\circ) = -19.1 \text{ m}$$

$$d_{3y} = d_3 \sin \theta_3 = 11 \text{ m} \cdot \sin 45^\circ = 7.78 \text{ m}$$

Notice that  $d_{2y}$  is negative. As I mentioned in the previous section, a negative sign on a vector component indicates its direction. Here  $d_{2y}$  points down, so the component is negative. The magnitude of the  $y$ -component,  $R_y$ , of the resultant,  $\mathbf{R}$ , is the sum of these  $y$ -components.

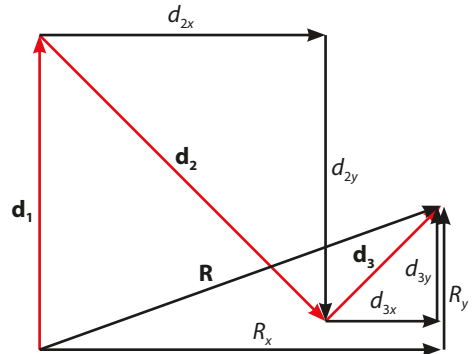


Figure 1.21. Three displacements, resultant, and all components. Note: Components  $d_{3y}$  and  $R_y$  are collinear, so they have been displaced slightly left and right to make  $d_{3y}$  visible.

$$R_y = 21.0 \text{ m} - 19.1 \text{ m} + 7.78 \text{ m} = 9.7 \text{ m}$$

Finally, we calculate the magnitude and direction of the resultant:

$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(26.9 \text{ m})^2 + (9.7 \text{ m})^2} = 28.6 \text{ m}$$

$$\theta_r = \tan^{-1} \frac{9.7}{26.9} = 19.8^\circ$$

Rounding these to two significant digits yields the same results as in the graphical solution to this example problem.



One final point remains for this section. Recall from your study of trigonometry that the range of the inverse tangent function is  $-\pi/2$  to  $\pi/2$ , or  $-90^\circ$  to  $90^\circ$ . This means that when you compute the angle of a vector pointing into the second or third quadrant, the inverse tangent function will not give you the angle you require without adding an additional  $180^\circ$ .

To illustrate, a vector with horizontal and vertical components of  $-5 \text{ m}$  and  $3 \text{ m}$ , respectively, lies in the second quadrant and has an angle between  $90^\circ$  and  $180^\circ$ , but the inverse tangent function gives  $\tan^{-1}(-3/5) = -31^\circ$ . The correct angle is  $-31^\circ + 180^\circ = 149^\circ$ . Similarly, a vector with horizontal and vertical components of  $-5 \text{ m}$  and  $-3 \text{ m}$ , respectively, lies in the third quadrant and has an angle between  $180^\circ$  and  $270^\circ$ , but the inverse tangent function gives  $\tan^{-1}(3/5) = 31^\circ$ . The correct angle is  $31^\circ + 180^\circ = 211^\circ$ . When computing an angle with the inverse tangent function, *always work with a diagram of the vector in front of you and correct the results of the inverse tangent function so that the angle you give to the vector makes sense in light of its position in the coordinate system.*

### 1.4.6 Multiplying a Vector by a Scalar

The simplest computation involving vectors is when a vector is multiplied by a scalar. When a vector is multiplied by a positive scalar, the effect is to change the vector's magnitude; the direction is unaffected. To illustrate, consider the following velocity vector:

$$\mathbf{v}: v = 25.0 \text{ m/s}, \theta_v = 67^\circ$$

If we multiply this velocity by a scalar quantity such as a mass  $m = 5.00 \text{ kg}$ , we would obtain a new vector with the same direction, a different magnitude, and a different set of units. As it turns out, the product of mass and velocity is called the *momentum*, our topic in Chapter 6. The symbol for the vector momentum is  $\mathbf{p}$ . So using this symbol, we calculate the magnitude and direction of the momentum vector in this illustration as follows:

$$\mathbf{p} = m \cdot \mathbf{v}$$

$$p = mv = (5.00 \text{ kg}) \cdot (25.0 \text{ m/s}) = 125 \text{ kg}\cdot\text{m/s}$$

$$\theta_p = \theta_v = 67^\circ$$

The two vectors  $\mathbf{v}$  and  $\mathbf{p}$  are shown in Figure 1.22.

Multiplying a vector by a negative scalar also occurs in some important topics in physics, such as forces on charged particles in electric fields. The negative sign on the scalar reverses the direction of the product vector you

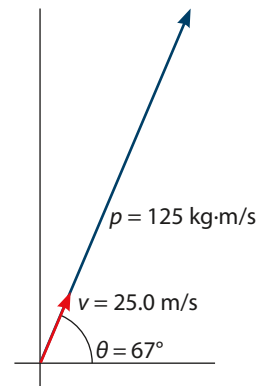


Figure 1.22. Vector  $\mathbf{p}$  is obtained by multiplying vector  $\mathbf{v}$  by a positive scalar.

obtain from the multiplication. In the previous two sections, we have seen that negative signs are used to indicate a negative direction. As illustrated in Figure 1.23, when multiplying by a negative scalar, the effect of the negative sign is to reverse the direction of the product vector relative to the original vector in the multiplication—an angular difference of  $180^\circ$ . To illustrate, consider a vector  $\mathbf{A}$  with a magnitude of  $A = 2.25$  units and a direction of  $105^\circ$ . If we multiplied this vector by the scalar quantity  $-1.70$  units, the new vector  $\mathbf{B}$  would have a magnitude of  $B = 3.83$  (with appropriate units) and a direction of  $-75^\circ$ .

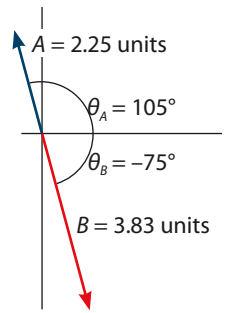


Figure 1.23. Vector  $\mathbf{B}$  is obtained by multiplying vector  $\mathbf{A}$  by a negative scalar.

### 1.4.7 Vector Multiplication

As mentioned in Section 1.4.2, you will probably skip this section until you need to learn vector multiplication methods (Chapters 4 and 5). Of course, if you just can't wait to learn how vector multiplication works, then read on.

We must pause here and remind ourselves that we use vector methods in physics because we are modeling nature. Some quantities in nature are directed, such as forces and velocities. Some quantities in nature are not directed, such as mass and energy. When we multiply two quantities together to get a third quantity, the third quantity itself can either be a directed quantity—a vector—or a quantity without any direction—a scalar. This implies that we need two different ways to perform the multiplication of two vectors—one that results in a scalar and one that results in a vector. This is exactly what we do have. The two vector multiplication operations are called the *dot product* and the *cross product*. The dot product results in a scalar; the cross product results in a new vector.

#### The Dot Product

The dot product of vectors  $\mathbf{A}$  and  $\mathbf{B}$  is defined as follows:

$$\mathbf{A} \cdot \mathbf{B} = |\mathbf{A}||\mathbf{B}|\cos \theta = AB\cos \theta \tag{1.1}$$

where  $\theta$  is the angle between  $\mathbf{A}$  and  $\mathbf{B}$ . The product of this operation is a scalar, and the dot product is often referred to as the *scalar product*. The value of the scalar product is  $|\mathbf{A}||\mathbf{B}|\cos \theta$ .

To illustrate the meaning of the dot product, consider the two vectors,  $\mathbf{A}$  and  $\mathbf{B}$ , shown in Figure 1.24. The vectors are separated by the angle  $\theta$ . Imagine vector  $\mathbf{B}$  as the hypotenuse of its own right triangle with the angle  $\theta$ . As illustrated on the left side of Figure 1.25, the product  $B\cos \theta$  is the magnitude of the component of  $\mathbf{B}$  that is parallel to  $\mathbf{A}$ . So the product  $AB\cos \theta$  is the product of the magnitudes of two vectors,  $\mathbf{A}$  and the component of  $\mathbf{B}$  that points in the same direction as  $\mathbf{A}$ .

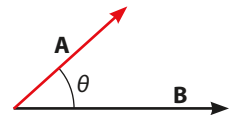
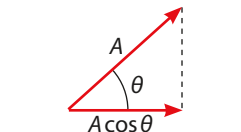
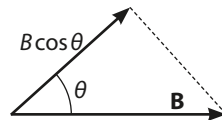


Figure 1.24. Vectors  $\mathbf{A}$  and  $\mathbf{B}$  separated by an angle  $\theta$ .

Now consider the right side of Figure 1.25. Imagine vector  $\mathbf{A}$  as the hypotenuse of its own right triangle with the angle  $\theta$ . The product  $A\cos \theta$  is the magnitude of the component of  $\mathbf{A}$  that is parallel to  $\mathbf{B}$ . So the product  $AB\cos \theta$  is the product of the magnitudes of two vectors,  $\mathbf{B}$  and the component of  $\mathbf{A}$  that points in the same direction as  $\mathbf{B}$ .



It does not matter which way you think about the result from the scalar product—as the product of  $A$  and the magnitude of the component of  $\mathbf{B}$  parallel to  $\mathbf{A}$ , or  $B$  and the magnitude of the component of  $\mathbf{A}$  parallel to  $\mathbf{B}$ . Either way, the scalar product computes the product of two magnitudes

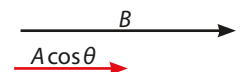
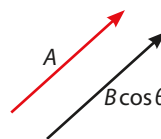


Figure 1.25. Two ways of interpreting the dot product  $\mathbf{A} \cdot \mathbf{B}$ .

that point in the same direction. The way you think about it when you are working out a problem depends on the problem.

A common example of the use of the dot product in physics is in computations involving *work*. In physics, mechanical *work* ( $W$ ) is defined as a process of energy transfer in which energy is transferred from one object to another by means of a force ( $F$ ) from the first object acting on the second object and pushing it through a given distance ( $d$ ). You may have seen the definition of work as

$$W = F \cdot d$$

Energy is measured in joules (J), one joule being equal to one newton-meter ( $\text{N} \cdot \text{m}$ ). Since work is a form of energy, the units for work are also joules. The formula above applies when the force on an object and the direction the object moves point in the same direction. However, more generally, forces are vectors and distances with directions are displacements, which are also vectors. Work is energy, and energy is a scalar. The product of force and distance is really a scalar product. Thus, from Equation (1.1) the more general definition of work is

$$W = \mathbf{F} \cdot \mathbf{d} = |\mathbf{F}||\mathbf{d}| \cos \theta = Fd \cos \theta$$

#### ▼ Example 1.4

Figure 1.26 shows a carton on the floor being pulled by a force of 75 newtons (75 N). The force, which could be due to a rope or cable attached to the carton, is being applied at an angle of  $35^\circ$  relative to the floor. Determine the amount of work done on the carton if the carton moves along the floor a distance of 0.50 m.

Using the floor as a reference for angles, the two vectors in this problem are:

$$\mathbf{F}: 75 \text{ N}, \triangle 35^\circ$$

$$\mathbf{d}: 0.50 \text{ m}, \triangle 0^\circ$$

The work is

$$W = Fd \cos \theta = (75 \text{ N}) \cdot (0.50 \text{ m}) \cdot \cos 35^\circ = 31 \text{ N} \cdot \text{m} = 31 \text{ J}$$

Since the motion is in the same direction as the displacement  $\mathbf{d}$ , it is useful to think of the scalar product in this problem as the product of  $d$  and the magnitude of the component of  $\mathbf{F}$  that points in the same direction as  $\mathbf{d}$ ,  $F \cos \theta$ . Only the horizontal component of  $\mathbf{F}$  contributes to the amount of work done. The vertical component of  $\mathbf{F}$  points up, and since the carton does not move any distance vertically, the vertical component of  $\mathbf{F}$  does not contribute to the work done.



#### The Cross Product

The result of a cross product calculation is a new vector, so the cross product is often referred to as the *vector product*. Explaining this calculation requires describing two separate rules, one for the magnitude and one for the direction. We will address the magnitude first.

The magnitude of the cross product of two vectors  $\mathbf{A}$  and  $\mathbf{B}$  is defined as follows:

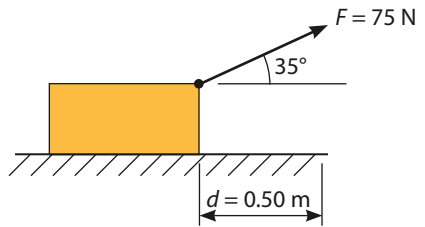


Figure 1.26. A force of magnitude  $F$  acting on an object and pulling it a distance  $d$ .



$$|\mathbf{A} \times \mathbf{B}| = |\mathbf{A}||\mathbf{B}|\sin\theta = AB\sin\theta \tag{1.2}$$

where, again,  $\theta$  is the angle between  $\mathbf{A}$  and  $\mathbf{B}$ . As before in the dot product description, consider Figure 1.24, with two vectors separated by an angle  $\theta$ . The entire description above of the two ways to think about the dot product applies here as well, only this time we are dealing with the sine of the angle instead of the cosine of the angle. This is illustrated in Figure 1.27. As you can see from the figure, regardless of how you think about it, the magnitude of the cross product is the product of the magnitude of one vector and the magnitude of the component of a second vector that is perpendicular to the first vector.

There is a mathematically sophisticated way to determine the direction of the vector product, but we will not address it here. That method is useful for physics and engineering problems in three dimensions, but since the problems we treat in this text will all be in two dimensions, the more sophisticated method entails more complexity than we need. If you take courses in physics or engineering in college you will see it there. For our purposes, the so-called *right-hand rule* works well enough.

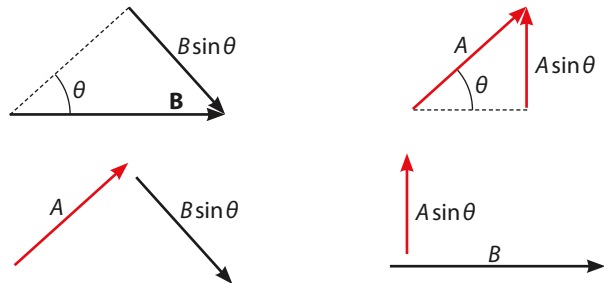


Figure 1.27. Two ways of interpreting the magnitude of the cross product  $\mathbf{A} \times \mathbf{B}$ .

To begin, the direction of the vector product is perpendicular to the plane formed by the two vectors being multiplied. This means that there are two possibilities for the direction. Mathematically, we would say that the cross product is not commutative like the dot product is. In fact, the direction of  $\mathbf{A} \times \mathbf{B}$  is exactly the opposite of the direction of  $\mathbf{B} \times \mathbf{A}$  (although their magnitudes are the same).

Figure 1.28 illustrates the use of the right-hand rule to determine the direction of  $\mathbf{A} \times \mathbf{B}$ . (Hold your right hand up in the air and practice the motion while you read this!) Flatten your hand and stick your thumb out straight to the side. Now imagine placing vector  $\mathbf{A}$  in the palm of your hand with the tip of the vector at your fingertips and the tail at the heel of your hand. Orient your hand so that your right palm can push or rotate  $\mathbf{A}$  toward vector  $\mathbf{B}$ , as indicated by the yellow arrows. (In the parlance of physics, this is called “crossing  $\mathbf{A}$  into  $\mathbf{B}$ .”) When your hand is oriented correctly, your thumb indicates the direction of the vector product  $\mathbf{A} \times \mathbf{B}$ . The direction of the cross product  $\mathbf{A} \times \mathbf{B}$  is perpendicular to both  $\mathbf{A}$  and  $\mathbf{B}$ .

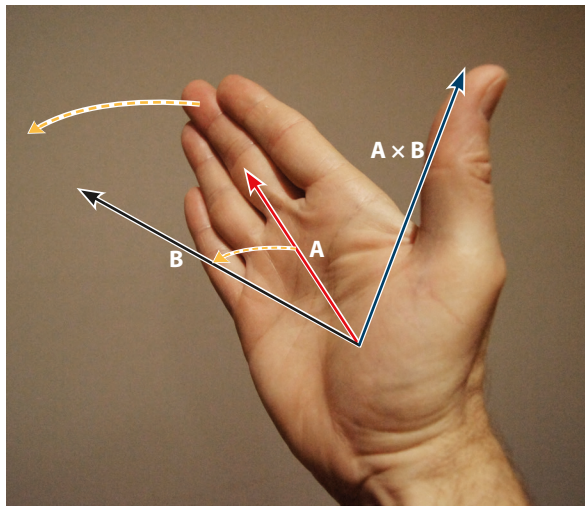


Figure 1.28. Using the right-hand rule to determine the direction of the cross product  $\mathbf{A} \times \mathbf{B}$ .

Figure 1.29 shows the result of  $\mathbf{B} \times \mathbf{A}$ . When  $\mathbf{B}$  is crossed into  $\mathbf{A}$ , the vector product points in the opposite direction. I didn't show a hand in Figure 1.29, but you can see that if a hand were there we would be looking at the back of the hand with its thumb pointing down and the vectors on the other side of it out of view. (Use your own hand to convince yourself of this.)



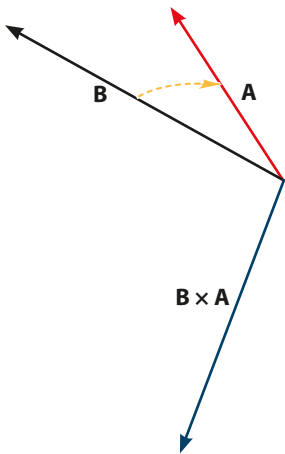


Figure 1.29. The cross product of  $\mathbf{B} \times \mathbf{A}$  points in the opposite direction from that of  $\mathbf{A} \times \mathbf{B}$ .

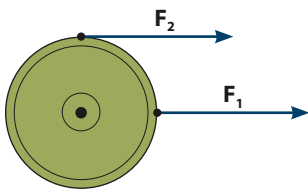


Figure 1.30. Force  $\mathbf{F}_2$  tends to cause the disk to rotate; force  $\mathbf{F}_1$  does not.

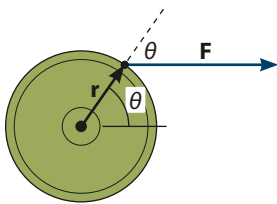


Figure 1.31. Force  $\mathbf{F}$  applied at a location described by displacement vector,  $\mathbf{r}$ .

A common application of the vector product is in the computation of *torque*. A torque is the result of a force that acts on an object in a way that tends to cause rotation. As indicated by Figure 1.30, force  $\mathbf{F}_1$  might cause the disk to move to the right if it was free to move, but it would not tend to cause the disk to rotate about its axle. However, force  $\mathbf{F}_2$  acts on the disk in a way that tends to cause rotation on its axle. If the disk is free to rotate and force  $\mathbf{F}_2$  is applied to the disk, then rotation will occur. The chain on a bicycle pulls on the sprocket just like force  $\mathbf{F}_2$  in the figure.

Torque is a vector quantity. The disk in Figure 1.30 can rotate in two different directions. If force  $\mathbf{F}_2$  pulls to the right as shown, a clockwise torque is produced and the disk rotates clockwise. If  $\mathbf{F}_2$  were pulling to the left, then a counterclockwise torque would be produced and the disk would rotate counterclockwise. The two directions correspond to the two directional possibilities for the vector resulting from a vector product.

Figure 1.31 shows a force  $\mathbf{F}$  applied to the rim of a disk at an arbitrary location. The displacement vector from the axle of the disk to the point where  $\mathbf{F}$  is attached is indicated as  $\mathbf{r}$ , and  $\mathbf{r}$  makes an angle  $\theta$  with the horizontal. The torque caused by a force depends on three variables: the strength of the force, the direction of the force, and the distance the force is applied from the center of rotation (the axle in this case). You probably can guess from your own experience that a force applied farther from the center of rotation produces a larger torque. This is why a multi-speed bicycle is easier to pedal if the chain is on the larger gear on the back sprocket than when it is on a smaller gear. It is also why a large steering wheel on a car or truck is easier to turn than a small steering wheel.

The torque,  $\boldsymbol{\tau}$ , produced by a force,  $\mathbf{F}$ , is given by the equation

$$\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F}$$

where  $\mathbf{r}$  is the displacement vector from the center of rotation to the point where the force is applied to the rotating object. The symbol for the torque variable,  $\tau$ , is the lower-case Greek letter *tau*, the *t* in the Greek alphabet. The angle used to compute this vector product is the angle between the lines the vectors are pointing along. In Figure 1.30, this is the angle between the dashed line and  $\mathbf{F}$ .

According to Equation (1.2), the magnitude of a torque is

$$\tau = rF \sin \theta$$

The direction of a torque is considered mathematically to lie *along the axis of rotation*. The direction it points is given by the right-hand rule, as illustrated in the following example.

▼ Example 1.5

Referring to back Figure 1.31, determine the torque produced by a force of 125 N if the radius of the disk is 6.25 cm and the angle is  $\theta = 55.0^\circ$ .

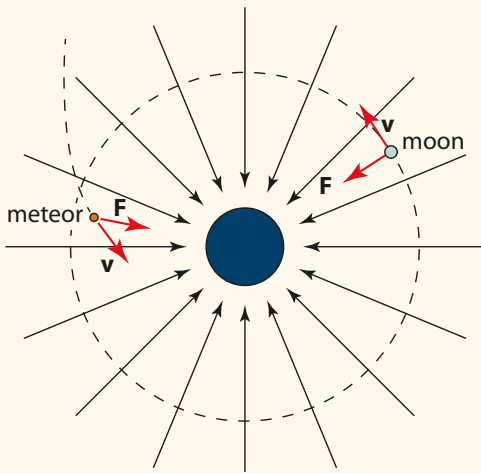
Converting the disk radius to MKS units, we have:

## Connections in Physics

## Vector Fields

Just as with velocities and forces, *fields* can be vector constructs. Gravitational fields, electric fields, and magnetic fields are all examples of vector fields. The effect of a field is to produce a force on certain types of objects. An electric field, for example, produces a force on any charged particle located in the field, and a gravitational field obviously exerts a force on a mass located in the field.

A vector field has a strength (magnitude) and direction at each point in space. These are represented by field lines—arrows—that indicate the direction of the field with arrowheads and the strength of the field by the spacing of the field lines. The closer the field lines are together, the stronger the field is.

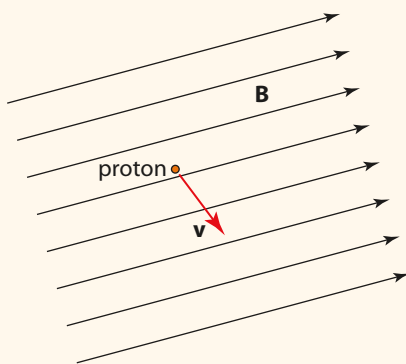


In a vector field, the force produced by the field is proportional to the strength of the field. In the diagram to the left, the black arrows are the field lines representing the gravitational field around the earth. Any object possessing mass in earth's gravitational field experiences a force directed toward the center of the earth.

Two objects are shown in the gravitational field, the moon and a meteor. The moon is in orbit, so its velocity vector points along its orbital path. The gravitational force the earth exerts on the moon is at right angles to the moon's velocity.

The meteor is shown entering the earth's gravitational field at an arbitrary angle. At any moment, its velocity also points in the direction it is moving, and the gravitational force on the meteor is directed toward the earth, just like the field lines.

You probably know that magnetic fields exert forces on ferrous materials and other magnets. As we will study much later, magnetic fields also exert forces on moving charges.



This is the only example we will study where the force exerted by a field is velocity-dependent. In fact, the force generated on a moving charge can be calculated using the cross product discussed in this chapter. The strength of a magnetic field is represented by the vector  $\mathbf{B}$  and the amount of charge present on a charged object (such as a proton) is denoted as the scalar  $q$ . The force on a moving charge in a magnetic field is

$$\mathbf{F} = q(\mathbf{v} \times \mathbf{B})$$

I will leave it to you to use the right-hand rule to determine the direction of  $\mathbf{F}$  in the figure and leave the answer for the exercises at the end of the chapter. After the cross product  $\mathbf{v} \times \mathbf{B}$  is calculated, the resulting vector must be multiplied by the scalar  $q$  to determine the force.

$$F = 125 \text{ N}$$

$$r = 6.25 \text{ cm} \cdot \frac{1 \text{ m}}{100 \text{ cm}} = 0.0625 \text{ m}$$

$$\theta = 55.0^\circ$$

From Equation (1.2), the magnitude of the torque is

$$\tau = rF \sin \theta = (0.0625 \text{ m}) \cdot (125 \text{ N}) \cdot \sin 55.0^\circ = 6.40 \text{ m} \cdot \text{N}$$

To determine the direction of the torque from the right-hand rule, you need to imagine sliding the displacement vector  $\mathbf{r}$  along the direction it points (the dashed line) until the tails of  $\mathbf{r}$  and  $\mathbf{F}$  are at the same spot, as illustrated in Figure 1.32. Then use your mental right hand to cross  $\mathbf{r}$  into  $\mathbf{F}$  to obtain the direction of the torque vector  $\boldsymbol{\tau}$ . (When first learning how to use the right-hand rule, it is wise to use your physical right hand as well, literally holding it up in the air and making the motions illustrated in Figure 1.28.) With the direction of  $\boldsymbol{\tau}$  determined, we can finally depict all three vectors in their proper places as shown in Figure 1.33.



As shown in Example 1.5, the units for torque are meter-newtons ( $\text{m} \cdot \text{N}$ , pronounced “meter newtons”). Although the units for torque are mathematically equivalent to the units for work ( $\text{N} \cdot \text{m}$ , or J, see Example 1.4), you must remember that we are using these units of measure to model the physical world, and in the world,  $\text{m} \cdot \text{N}$  and  $\text{N} \cdot \text{m}$  have entirely different physical interpretations. Work is a scalar; torque is a vector. So when we speak of these units, we are careful about the order in which the meters and newtons are spoken or written. The correct order for the units is indicated by the standard ways of writing the formulas for work and torque. In the work equation, the order is force before displacement and the unit is newton-meter; in the torque equation the order is displacement before force and the unit is meter-newton. The same care about order applies when torque is specified in the common USCS units of inch-pounds or foot-pounds. For torques, the length unit is written first.

## Chapter 1 Exercises

For all exercises, note that physical constants and unit conversion factors are found in Appendix A and inside the rear cover.

### SECTION 1.1

1. Explain why measurement is such a central concern in science.
2. Distinguish between matter, inertia, and mass.
3. Compare volume and mass.

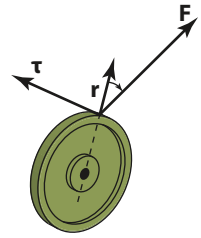


Figure 1.32. Mentally place the tails of  $\mathbf{r}$  and  $\mathbf{F}$  together to cross  $\mathbf{r}$  into  $\mathbf{F}$ .

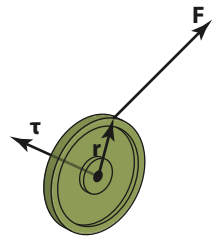


Figure 1.33. A 3-D view of  $\mathbf{r}$ ,  $\mathbf{F}$ , and  $\boldsymbol{\tau}$ .

4. Perform the following unit conversions using all prefixes and their conversion factors from memory. (Note: If you have not yet mastered the execution of unit conversions, you will find a tutorial on the subject in Appendix B.)

Convert This Quantity	Into These Units	Convert This Quantity	Into These Units
a. 35.4 mm	m	b. 76.991 mL	$\mu\text{L}$
c. 34.44 $\text{cm}^3$	L	d. 6.33 $\text{g}/\text{cm}^2$	$\text{kg}/\text{m}^2$
e. 9.35 $\text{m}/\text{s}^2$	$\text{mm}/\text{ms}^2$	f. 542.2 $\text{mJ}/\text{s}$	$\text{J}/\text{s}$
g. 56.6 $\mu\text{s}$	ms	h. 44.19 mL	$\text{cm}^3$
i. 532 nm	$\mu\text{m}$	j. 96,963,000 $\text{mL}/\text{ms}$	$\text{m}^3/\text{s}$
k. 295.6 cL	$\mu\text{L}$	l. 0.007873 $\text{m}^3$	mL
m. 8,750 $\text{mm}^2$	$\text{m}^2$	n. 87.1 $\text{cm}/\text{s}^2$	$\text{m}/\text{s}^2$
o. 15.75 $\text{kg}/\text{m}^3$	$\text{g}/\text{cm}^3$	p. 0.875 km	m
q. 16,056 MPa	kPa	r. 7,845 $\mu\text{A}$	mA

5. Distinguish between base and derived units.  
6. Describe the origin of the SI system of units.

#### SECTION 1.2

7. Describe the effect of uncertainty on scientific measurements.  
8. Write a paragraph distinguishing between accuracy and precision.  
9. Distinguish between random error and systematic error.  
10. Explain the meaning of the standard deviation of a data set and how the standard deviation relates to uncertainty in measurements.

#### SECTION 1.3

11. Explain why scientists should avoid terms such as “proven” and “true” when describing scientific knowledge.  
12. Describe the ways we apprehend truth, and contrast these to the processes and goals of scientific inquiry.  
13. Explain the difference between truth and facts.  
14. Describe the role of theories in scientific research.  
15. I once read the statement, “No theory is true until it is proven.” Use our discussion of the nature and role of theories and scientific knowledge to respond to this claim.

#### SECTION 1.4

*Solve problems 16–19 using graphical methods.*

16. A Boy Scout must go on a hike of 33 km to qualify for the hiking merit badge. For his hike, the boy decides to walk to a friend’s house as follows: due east from his house for 9.5 km, then due southeast for 7.1 km. From there he will return home. What is the magnitude and direction of the displacement vector from the Scout’s house to his friend’s house?  
17. Determine the resultant vector from the sum of the following three displacements:  
 $\mathbf{d}_1 \quad d_1 = 10.5 \text{ cm}, \theta_1 = -47^\circ$   
 $\mathbf{d}_2 \quad d_2 = 8.2 \text{ cm}, \theta_2 = -110^\circ$   
 $\mathbf{d}_3 \quad d_3 = 10.5 \text{ cm}, \theta_3 = 116^\circ$

18. Add the following vectors and determine the magnitude and direction of the resultant:

$$\mathbf{F}_1 \quad F_1 = 66.4 \text{ N}, \theta_1 = 37.5^\circ$$

$$\mathbf{F}_2 \quad F_2 = 59.8 \text{ N}, \theta_2 = -122^\circ$$

$$\mathbf{F}_3 \quad F_3 = 35.1 \text{ N}, \theta_3 = 131.5^\circ$$

19. The *ground velocity* of an airplane is the speed and direction of the plane relative to the ground. The pilot maintains a certain *air speed*, which is the speed of the plane relative to the air, but if there is wind, the velocity of the wind must be taken into account to determine the plane's ground velocity. The ground velocity is calculated as the vector sum of the velocity of the wind and the plane's air speed and direction. A plane flies with an air speed of 275 km/hr at a bearing of  $276^\circ$ . The wind velocity is 45 km/hr at a bearing of  $185^\circ$ . Determine the plane's ground velocity. (Note: The term *bearing* is used in navigation to indicate an angle measured clockwise from due north.)

20. Compute the following. (The units in the third problem are fabrications; they have no actual physical meaning.)

a.  $\mathbf{p} = m \cdot \mathbf{v}$ , where  $m = 4.31 \times 10^{-26}$  kg,  $v = 2.994 \times 10^6$  m/s, and  $\theta_v = 23^\circ$

b.  $\mathbf{F} = q \cdot \mathbf{E}$ , where  $q = -2.25 \times 10^{-6}$  C,  $E = 19.95$  V/m, and  $\theta_E = 161^\circ$

c.  $\mathbf{R} = n \cdot \mathbf{s}$ , where  $n = -15.5$  xg,  $s = 57.9$  jd, and  $\theta_s = -135^\circ$

21. Determine the magnitude of the vector represented by the following pairs of components:

a.  $\mathbf{v}_1$ , where  $v_{1x} = 25$  m/s,  $v_{1y} = 14$  m/s

b.  $\mathbf{v}_p$ , where  $v_{fx} = -24.765$  cm/s,  $v_{fy} = -67.001$  cm/s

c.  $\mathbf{d}$ , where  $d_x = 1.00 \times 10^{-3}$  cm,  $d_y = -6.77 \times 10^{-4}$  cm

d.  $\mathbf{F}_1$ , where  $F_{1x} = -355$  N,  $F_{1y} = 865$  N

e.  $\mathbf{a}$ , where  $a_x = -2.124$  m/s<sup>2</sup>,  $a_y = 3.910$  m/s<sup>2</sup>

f.  $\mathbf{E}$ , where  $E_x = -0.0091$  V/m,  $E_y = -0.0104$  V/m

22. Determine the direction of the vector represented by each of the pairs of components in the previous item.

23. Add the following vectors and determine the magnitude and direction of the resultant:

$$\mathbf{v}_1: \quad v_1 = 45.6 \text{ cm/s}, \theta_1 = 123^\circ$$

$$\mathbf{v}_2: \quad v_2 = 98.1 \text{ cm/s}, \theta_2 = 16.1^\circ$$

24. Add the following vectors and determine the magnitude and direction of the resultant:

$$\mathbf{a}_1: \quad a_1 = 45.0 \text{ m/s}^2, \theta_1 = 90.0^\circ$$

$$\mathbf{a}_2: \quad a_2 = 100.7 \text{ m/s}^2, \theta_2 = 235^\circ$$

25. To travel from Austin, Texas to Atlanta, Georgia a plane must fly 1,319 km at a bearing of  $69.5^\circ$ . To travel from St. Louis to Atlanta a plane must fly 753 km at a bearing of  $134.0^\circ$ . Determine the distance and bearing a plane must fly to travel from Austin to St. Louis. (Recall, the term *bearing* is used in navigation to indicate an angle measured clockwise from due north.)

26. An automated drilling tool begins its operation with the drill bit at the origin of a coordinate system. We will call the origin Point #1. After drilling a hole there, the drill undergoes a displacement of 13.00 cm,  $\triangle 75.00^\circ$  to arrive at Point #2. After drilling there, the drill undergoes a second displacement of 17.00 cm,  $\triangle 225.00^\circ$  to arrive at Point #3. After drilling there, the drill undergoes a third displacement of 4.50 cm,  $\triangle 0.00^\circ$  to arrive at Point #4.

Determine where Point #4 is relative to the starting point (i.e., the displacement vector from the origin to Point #4).

27. Electrical forces are present on an electron, as shown in Figure 1.34. Determine the resultant vector from the sum of these three forces.

28. Add the following vectors and determine the magnitude and direction of the resultant:

$$\mathbf{F}_1 \quad F_1 = 72.1 \text{ N}, \theta_1 = -117^\circ$$

$$\mathbf{F}_2 \quad F_2 = 73.0 \text{ N}, \theta_2 = 46.4^\circ$$

$$\mathbf{F}_3 \quad F_3 = 84.2 \text{ N}, \theta_3 = 141^\circ$$

29. Add the following vectors and determine the magnitude and direction of the resultant:

$$\mathbf{F}_1 \quad F_1 = 2,450 \text{ N}, \theta_1 = 25.0^\circ$$

$$\mathbf{F}_2 \quad F_2 = 1,965 \text{ N}, \theta_2 = -13.5^\circ$$

$$\mathbf{F}_3 \quad F_3 = 1,370 \text{ N}, \theta_3 = 175.1^\circ$$

$$\mathbf{F}_4 \quad F_4 = 2,009 \text{ N}, \theta_4 = -101.5^\circ$$

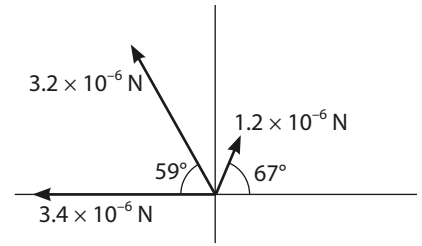


Figure 1.34. Vectors for exercise 27.

**SECTION 1.4.7**

30. Determine the following scalar products:

a.  $\mathbf{A} \cdot \mathbf{B}$ , where  $A = 14.6 \text{ N}$ ,  $\theta_A = 0.00^\circ$ ,  $B = 16.0 \text{ m}$ ,  $\theta_B = 52.8^\circ$

b.  $W = \mathbf{F} \cdot \mathbf{d}$ , where  $F = 9.21 \times 10^4 \text{ N}$ ,  $\theta_F = -167^\circ$ ,  $d = 4.021 \times 10^{-5} \text{ m}$ ,  $\theta_d = -31.1^\circ$

c.  $U = -\mathbf{p} \cdot \mathbf{E}$ , where  $p = 0.0258 \text{ m} \cdot \text{C}$ ,  $\theta_p = 150.7^\circ$ ,  $E = 6.02 \times 10^4 \text{ N/C}$ ,  $\theta_E = -119.0^\circ$

31. Determine the following vector products. In each case, specify the direction of the product as pointing into the page or pointing out of the page.

a.  $\mathbf{A} \times \mathbf{B}$ , where  $A = 53.2 \text{ m}$ ,  $\theta_A = 25.1^\circ$ ,  $B = 16.0 \text{ N}$ ,  $\theta_B = -48.2^\circ$

b.  $\mathbf{B} \times \mathbf{A}$ , where  $A = 53.2 \text{ m}$ ,  $\theta_A = 25.1^\circ$ ,  $B = 16.0 \text{ N}$ ,  $\theta_B = -48.2^\circ$

c.  $\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F}$ , where  $r = 0.0234 \text{ m}$ ,  $\theta_r = 225^\circ$ ,  $F = 6.18 \times 10^{-5} \text{ N}$ ,  $\theta_F = 122^\circ$

d.  $\boldsymbol{\tau} = \mathbf{p} \times \mathbf{E}$ , where  $p = 1.75 \times 10^{-3} \text{ m} \cdot \text{C}$ ,  $\theta_p = 11.9^\circ$ ,  $E = 4.96 \times 10^5 \text{ N/C}$ ,  $\theta_E = -215.6^\circ$

32. For the magnetic field example discussed at the bottom of page 33, assume  $\mathbf{v} = 750 \text{ m/s}$ ,  $\hat{\Delta} -55^\circ$  and  $\mathbf{B} = 0.15 \text{ T}$ ,  $\hat{\Delta} 22^\circ$ . Determine  $\mathbf{F}$  for a proton and for an electron. (The unit for  $\mathbf{B}$  is the tesla [T] and the unit for  $q$  is the coulomb [C], but these are both MKS units.)



## Chapter 2

---

### Uniform Motion



A stream of water follows a parabolic path, illustrating *projectile motion*. In this chapter we will explore a simple model of this type of motion and apply the methods of vector analysis to the solution of various problems.



## Objectives for Chapter 2

---

After studying this chapter and completing the exercises, you should be able to do each of the following tasks, using supporting terms and principles as necessary.

### SECTION 2.1

1. Define *physics*.

### SECTION 2.2

2. Define *kinematics*, *dynamics*, and *rectilinear motion*.
3. Define *uniform motion*, *velocity*, and *acceleration* in words.
4. Define *average velocity* and *uniform acceleration* with equations.
5. Calculate average velocity and uniform acceleration using the definitions of these terms.
6. Interpret graphs of displacement vs. time and velocity vs. time, including composite graphs representing several separate intervals of uniform motion.
7. Use proper problem-solving methods.
8. Use the equations of kinematics to solve vector problems in one dimension (rectilinear motion).

### SECTION 2.3

9. Define *projectile motion*.
10. Describe the assumptions implicit in solving typical problems in projectile motion.
11. Use vector analysis and the kinematics equations from memory to solve projectile motion problems

### SECTION 2.4

12. Define *relative motion*.
13. Draw diagrams showing the resultant of two vectors and the resolution of a vector into rectangular components.
14. Use vector analysis and other trigonometric principles to solve relative velocity problems.

## 2.1 How Physics Is Organized

*Physics* is usually described as the study of matter and energy and their interactions. In this study, we constantly encounter ideas that are fascinating, mystifying, stupefying, or all of the above, and this is why folks like me simply can't get enough of this wonderful subject.

The study of physics is often subdivided into several major branches. These include *mechanics*, *electricity and magnetism*, *thermodynamics*, and *quantum mechanics*. Beginning with this chapter, the next few chapters in this text all focus on the principles and problems in mechanics. The study of mechanics always begins with the topic of motion. After that we will take up the topic of energy before returning to motion again to address some of the more advanced concepts.

## 2.2 Rectilinear Motion

### 2.2.1 The Terminology of Motion

The study of motion can be subdivided into *kinematics* and *dynamics*. Kinematics is the subject of this chapter; dynamics is the subject of Chapter 3. Kinematics is the study of motion purely from the definitions of velocity and acceleration, without regard to the forces causing the acceleration. In dynamics, we study motion from the standpoint of the forces involved and the accelerations that result. Together, kinematics and dynamics are part of the more general subject known as *mechanics*.

The simplest form of motion is motion in a straight line, or movement in one dimension. This subject is called *rectilinear motion* and is our first topic in this chapter. After mastering problems in one dimension, we will be ready for motion in two dimensions, which includes *projectile motion*. That's when things start to get really interesting (and the problems start to get more challenging).

As it happens, if we start with just a few equations defining velocity and acceleration we can derive a set of equations we can use to calculate any time, displacement, velocity, or acceleration associated with an object undergoing uniform acceleration.

### 2.2.2 Coordinate Systems

For all problem solving in mechanics, we choose a convenient coordinate system for a given problem and define quantities in terms of this coordinate system. To be consistent, once a specific direction is identified as the positive direction, all the variables in the problem are positive in the same direction. For problems involving motion in two dimensions, the usual coordinate system to use is the standard  $x$ - $y$  system, with the positive  $x$ -axis pointing to the right and the positive  $y$ -axis pointing up. However, there are times when it is more convenient to choose a different coordinate system. I will frequently comment on the coordinate system I am using for a given example.

### 2.2.3 Velocity and Acceleration

Figure 2.1 shows a graph of displacement vs. time for an object moving at a constant velocity, which means a constant speed in a straight line. As you see, the curves on this graph are straight lines representing specific velocities. Velocity is defined as an object's change in displacement per interval of time, or

$$\mathbf{v} = \frac{\Delta \mathbf{d}}{\Delta t} = \frac{d_f - d_i}{t_f - t_i} \quad (2.1)$$

The subscripts  $f$  and  $i$  indicate variable values at the end of the interval ( $f$ , or final) and at the beginning of the interval ( $i$ , or initial). Since we are working in one dimension, we can also use the variable  $x$  to represent the object's position. In this case Equation (2.1) would be written

$$\mathbf{v} = \frac{\Delta \mathbf{x}}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}$$

This expression makes it clear that the velocity of an object is equal to the slope of the line on the displacement vs. time graph representing its motion. As indicated in the center of the graph in the figure, if an object travels from  $d = 4$  m to  $d = 6$  m in the time interval from  $t = 1$  s to  $t = 1.5$  s, the slope of the line represents a velocity of

$$\mathbf{v} = \frac{\Delta \mathbf{d}}{\Delta t} = \frac{6 \text{ m} - 4 \text{ m}}{1.5 \text{ s} - 1 \text{ s}} = \frac{2 \text{ m}}{0.5 \text{ s}} = 4 \frac{\text{m}}{\text{s}}$$

Steeper curves on the graph indicate higher velocities. A horizontal line on this graph represents an object at the same position at all times, which means the object is at rest ( $v = 0$ ).

Again, we are presently discussing motion in one dimension. You should think in terms of an object moving on a horizontal track marked off like a number line, with positive positions right of zero and negative positions left of zero. Even in one dimension, the directional nature of the displacement and velocity vectors is relevant: displacement can be positive or negative because an object can be on either side of zero. Velocity can be either positive or negative because an object can be moving in a positive direction (its position is getting more and more positive) or in a negative direction (its position is getting more and more negative).

Figure 2.2 again shows a graph of displacement vs. time, this time with three different curves on the graph. The curves have different slopes representing three different velocities, two of which are negative. Four locations are marked on the curves; each of these locations represents an object at a specific position moving in a specific direction at a specific speed.

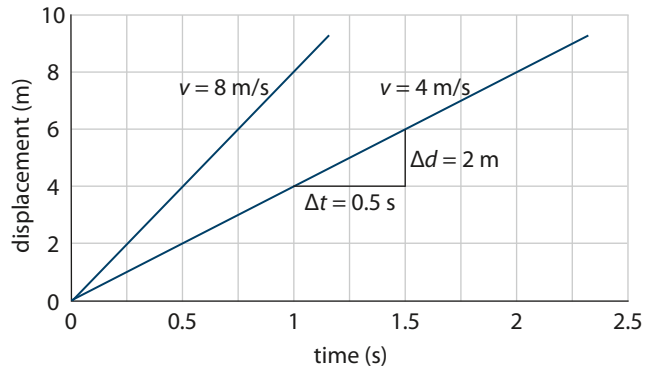


Figure 2.1. The velocity of an object moving at constant velocity is the slope of the line on a graph of displacement vs. time.

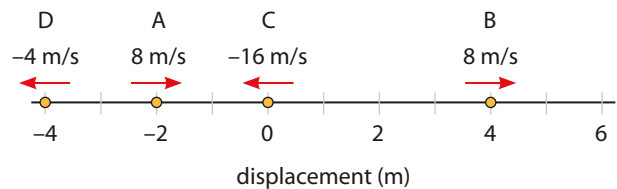
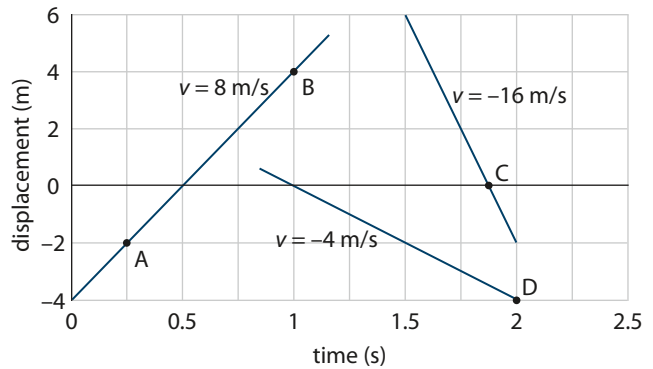


Figure 2.2. Comparison between points on velocity curves on a displacement vs. time graph and objects moving in one dimension.

Note that the curve containing points A and B *does not* represent an object “going up,” and the other two curves containing points C and D do not represent objects “going down.”

What the curves *do* represent is illustrated by the four objects on the number line at the bottom of the figure. On the number line, the velocity of each object is labeled—each object is moving at a specific speed, and each object is moving in a positive direction (to the right) or a negative direction (to the left), as indicated by both the sign on the velocity value and the direction shown by the red arrow. As one illustration, point D in the graph represents an object at the position  $x = -4$  m moving to the left at a velocity of  $v = -4$  m/s.

So far in this section, we have only discussed objects moving with constant velocity. However, if you have studied Newton’s Laws of Motion before, you know that when a net force is present on an object the object will not move at a constant velocity but will instead *accelerate*. When an object is accelerating, its velocity is changing. In this chapter, we are considering *uniform motion*, which means one of two things is going on:

1. An object is moving in a straight line at a constant speed, which means that the velocity is constant and the object is not accelerating (the case considered in Figures 2.1 and 2.2).
2. An object is accelerating *uniformly*, which means its velocity is changing at a constant rate in a certain direction.<sup>1</sup>

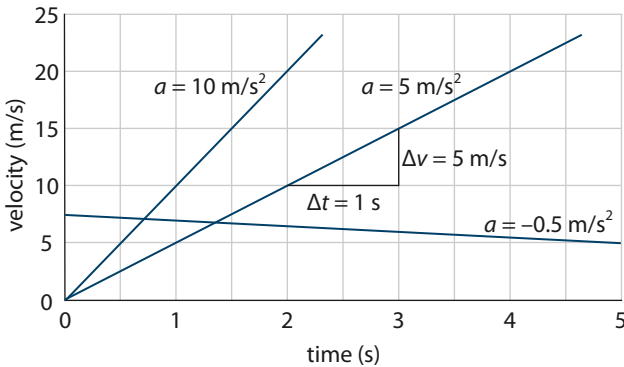


Figure 2.3. The acceleration of an object accelerating uniformly is the slope of the line on a graph of velocity vs. time.

Figure 2.3 shows three different curves representing velocity vs. time for an object accelerating uniformly. When acceleration is uniform, the velocity changes by the same amount in any equal interval of time. As you see, straight lines on this graph represent specific accelerations. Again, nothing on this graph represents anything moving up or down; the curves on the graph represent objects speeding up or slowing down at different rates. The steeper an upward slope, the faster an object’s velocity is increasing. The steeper a negative slope, the

faster an object is slowing down. A horizontal line on this graph represents an object moving at a constant speed.

Acceleration is defined as the change in velocity per interval of time, or

$$a = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{t_f - t_i} \tag{2.2}$$

As indicated in the center of the graph, if an object’s velocity changes from 10 m/s to 15 m/s in the time interval from  $t = 2$  s to  $t = 3$  s, the slope of the line represents an acceleration of

$$a = \frac{\Delta v}{\Delta t} = \frac{15 \frac{\text{m}}{\text{s}} - 10 \frac{\text{m}}{\text{s}}}{3 \text{ s} - 2 \text{ s}} = \frac{5 \frac{\text{m}}{\text{s}}}{1 \text{ s}} = 5 \frac{\text{m}}{\text{s}^2}$$

<sup>1</sup> Rotating objects can also accelerate uniformly, but we will not consider rotation until Chapter 7.

You do not need to be taking calculus to use this text; none of the mathematics required for solving the problems goes beyond standard topics in precalculus. However, many—if not most—of the students who take physics as high school seniors take calculus concurrently. This opens up the possibility for some enriching connections to be made between the physics and calculus-based math. So for the value of those who are taking calculus, these blue boxes will point out some important connections.

## Connections to Calculus

## Derivatives

By limiting the discussion in this chapter to uniform acceleration—the linear variation of velocity—we are able to define velocity and acceleration as shown in Equations (2.1) and (2.2). In the more general case, acceleration does not have to be uniform; the acceleration of a moving object may be defined by any continuous function of time.

By evaluating the limit of  $\Delta x/\Delta t$  as  $\Delta t$  goes to zero, we arrive at the rate of change of  $x$ , the time derivative of  $x$  for any continuous position function:

$$\lim_{\Delta t \rightarrow 0} \left( \frac{\Delta x}{\Delta t} \right) = \frac{dx}{dt}$$

But the rate of change of  $x$  is simply the object's velocity, so

$$v = \frac{dx}{dt}$$

Evaluating the derivative at a specific time gives the instantaneous velocity at that time. Likewise, we can evaluate the limit of  $\Delta v/\Delta t$  as  $\Delta t$  goes to zero, the time derivative of  $v$ :

$$\lim_{\Delta t \rightarrow 0} \left( \frac{\Delta v}{\Delta t} \right) = \frac{dv}{dt}$$

But the rate of change of  $v$  is simply the object's acceleration, so

$$a = \frac{dv}{dt}$$

Consider the case where the position of an object varies quadratically. Using  $k_1$ ,  $k_2$ , and  $k_3$  as coefficients, the generic quadratic function is  $x = k_1 t^2 + k_2 t + k_3$ . The velocity function is then  $v = dx/dt = 2k_1 t + k_2$ , which is a linear function. The acceleration is  $a = dv/dt = 2k_1$ , which is a constant. This is the case of uniform acceleration which we are considering in this chapter. Since  $a = 2k_1$ , the coefficient  $k_1$  is equal to  $a/2$ . Inserting this into the equation for  $v$ , we have  $v = (a/2)t + k_2$ . But this is a linear function with an initial value (the value at  $t = 0$ ) of  $k_2$ . Thus,  $k_2$  is the initial velocity of the object, which we denote as  $v_i$  or  $v_0$ . The constant  $k_3$  turns out to be the initial position of the object, which we denote as  $x_i$  or  $x_0$ . Replacing the generic  $k$  coefficients with these values, we have

$$x = \frac{1}{2}at^2 + v_0 t + x_0$$

As we will see in Section 2.2.4, if we set the position at  $t = 0$  to be  $x = 0$ , then  $k_3 = 0$  and the position function becomes

$$x = \frac{1}{2}at^2 + v_0 t$$